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## AUTOMATIC LEM MISSION STUDY [U]

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GRUMMAN AIRCRAFT ENGINEERING CORPORATION

ENG. 345

# PREFACE

This report presents the results of the redirected "automatic LEM mission" study defined in references 3 and 7. The ground rules for this effort differ markedly from those used for the original study of completely automatic LEM. The revised ground rules allow the crew to perform those functions which, as indicated in the results of the initial study (Ref. 9), would cause substantial increases in LEM weight and complexity if automatized. These functions include, for example, the switching required to activate and shutdown systems and the alignment of the inertial equipment which involves optical sightings and insertions of initial conditions into the computer.

## ABSTRACT

This report is intended to satisfy the requirements for the automatic LEM study. This study was undertaken in accordance with the requirements of Item C-3 of LEM Engineering Memo L250-M03-2 by C. W. Rathke, entitled "Technical Priority Efforts, Prime Responsibility for". This memo expressed the results of an agreement between GAEC and NASA on the LEM work effort. The study covered in this report was performed under the revised ground rules referred to in the preface.

The study briefly investigates the automation of the complete lunar descent from the standpoint of the two sets of ground rules proposed. i.e., Completely automatic vs. crew participation but completely automatic vehicle attitude and trajectory control. The study then focuses on the automation of the hover to touchdown maneuver. The report investigates in detail the suitability of the application of specific guidance laws as applied to this maneuver, and its impact on LEM hardware. It is immediately determined that change in hardware is minimal and that the major problem is to determine guidance laws which will produce successful trajectories under a wide range of hover and target point conditions. Two laws are studied for this application. They are the Proportional Line of Sight Navigation Law and the Polynomial Law. The Proportional Law is studied in great detail and a Modified Proportional Law is introduced, because it was found that the Basic Proportional Law did not satisfy the zero initial hover velocity requirement. In addition the Basic Law had associated with it only a limited range of hover velocity conditions which would produce successful landings (as defined in Section 2.3.2) with good visibility. It is concluded that the Modified Proportional Law produces successful trajectories under a wide range of hover and target range point conditions and could well be applied to the automatic hover to touchdown maneuver. Although the Polynomial Law produces suitable trajectory shapes and terminal conditions, it appears that some improvement could be made by removal of the oscillatory attitude response which exists in the first half of the trajectory.

SUMMARY

This report represents a detailed description of the adaptation of the Proportional Navigation Line of Sight (LOS) Guidance Law for use in the automatic hover to touchdown maneuver. The information obtained from the study of this law above was sufficient to demonstrate the feasibility and equipment requirements for the automatic mission study. However, in addition some work has been performed on the Polynomial Guidance Law developed at GAEC. Due to the amount of time still required for completion of the effort on the Polynomial Law, it was not planned to discuss this law as applied to the automatic maneuver. However just as the rought draft of this report was completed fruitful results were obtained for the Polynomial Law and a memo summarizing these results was published (LMO-500-110, Ref. 16). Since the memo on the Polynomial Law is adequately descriptive and it was not desired to hold up the issuance of this report, it was felt that it would be adequate to use that memo as a reference only. An introductory description of the Polynomial Law was however included as Section four of this report to supplement the LMO-500-110. The referenced memo indicates that the automatic hover-to-touchdown phase would be feasible with the Polynomial Law as outlined. This conclusion was reached after successful touchdown conditions were obtained on several trajectory runs for various values of N and K (guidance law constants). Some typical trajectories are shown in the referenced memo for various gain constants. In addition successful touchdown conditions are indicated for a wide range of trajectories. However pitch attitude profiles as a function of time indicate that rather oscillatory attitude motions are produced by the Polynomial Law for the first half of the maneuver before attitude settling occurs. Pitch rates are in the vicinity of 10°/sec. which are on the borderline of acceptability from the crew systems viewpoint. The memo also determined that an LGC computation rate of 2 samples per second gives sufficiently frequent data for use by the Polynomial Law.

Included in LMO-500-110 are the state errors at touchdown which would result from the introduction of guidance system errors and initial condition uncertainties. The plots show that under reasonable conditions of component and initial errors, satisfactory terminal conditions are attainable.

As indicated previously, the main body of this report concentrates on the implications of the use of the Proportional Navigation Law for the automatic hover-to-touchdown maneuver. This report concludes that minimal changes to LEM basic design would be required since most functions which would be required for a manned but automatic hover to touchdown maneuver already exist in the LEM. This is true provided that monitoring, switching and navigation and guidance alignment

functions are performed by the crew. Therefore nominally automatic guidance and control of the vehicle for the hover to touchdown maneuver may be performed essentially without the further addition of equipment. However, it should be pointed out that changes in the computer program may be required to insert the guidance law equations, and to adjust the Proportional Guidance Law gain constants for the existing conditions at hover (target range, LEM hover velocity, etc.). Selection of the landing point by use of an existing LEM reticle may possibly be performed, but since this hardware and function is not yet defined, judgement on its applicability must be deferred.

First, the constraints imposed on the trajectory during descent and at touchdown are discussed. Then later, the results of the application of the law (in the form of trajectory data obtained from computer simulation runs) are compared to these constraints to determine the degree of success of the guidance law used. Constraints discussed include the vehicle attitude and attitude rates for the complete trajectory. In addition, landing point and horizon visibility is considered. Touchdown velocities and attitude conditions are also considered in the evaluation of the law.

The theory of the Basic Proportional Law is discussed and its major limitation delineated. This limitation concerns the inability to generate guidance commands if the vehicle velocity is zero at hover. This drawback is overcome by modifying the Proportional Law to include acceleration commands in the gravity term. It is pointed out that this modification tends to make the Basic Law more adaptive to initial velocity conditions since the modification term is made a function of initial velocity.

In order to understand the effects on the trajectory, the gain constants in the Basic Proportional Law are discussed in considerable detail. The bounds on the constants are indicated and the expected effect of their variations on the trajectory time, shape, $\Delta V$ consummed, max. vehicle attitude off vertical, and interaction with initial velocity are indicated. Finally, data is presented which summarizes the results of computer simulation runs employing the Basic Proportional Law. This data is shown to follow the theoretically expected pattern and sheds further light on the more complex relationships involved. An alternative implementation of the Basic Guidance Law is discussed which involves the concept of changing guidance law gain constants to best fit vehicle initial conditions to the required constraints. For example, K gain (a Proportional Law gain constant) may be varied with initial velocity and LEM distance to target to optimize $\Delta V$ consumption, touchdown angular velocity and attitude.

There is a fair range of successful trajectories available if the proper value of the K gain is chosen for each set of initial conditions. However, visibility exhibited by the Basic Proportional Law trajectories is not good at the higher initial velocities and for close range targets. In addition the $\Delta V$ budget is exceeded if initial velocity is too low in relation to target range.

A discussion on the results of computer runs employing the Modified Proportional Law is presented. The effects of varying the values of initial conditions with system gains on miss distance, $\Delta V$, pitch attitude and touchdown pitch rates and attitude are also presented. The Modified Proportional Law is shown to exhibit superior low hover approach velocity characteristics consuming less $\Delta V$ than the Basic Proportional Law. In addition it is shown that visibility is improved by use of the Modified Law for closer range targets and higher hover velocities. It was found that the number of acceptable trajectories obtainable by use of the Modified Law is increased over that obtained by the Basic Law with the same initial conditions. It is felt that sufficient flexibility exists in the Modified Law to enable its successful application to the automatic hover to touchdown maneuver.

## ACKNOWLEDGEMENT

The Systems Analysis and Integration Section was given the responsibility of responding for GAEC to the contractual requirement on performance of the Automatic Mission Study. As a result Systems Analysis outlined a study plan which was to be followed and submitted this in the form of a study request to the Dynamic Analysis Group at GAEC (Ref. 7).

At this time it is appropriate to give credit to the individuals who performed the analysis and were responsible for performance of computer runs. The information, results and conclusions presented here were based largely on the results of their work effort. Acknowledgements are given as follows:

A. Bierman (LEM Dynamic Analysis, Guidance Dynamics Group, GAEC) for his work on the Proportional Navigation Law. One of the more important accomplishments of his effort in this area was the development of a modification to the Basic Proportional Navigation Guidance Law which appears to greatly improve the vehicle performance for the hover to touchdown maneuver. In addition, he was responsible for adapting the Basic Proportional Navigation Law to the hover to touchdown maneuver to which it has never previously been applied (See References 5, 6, 12 and 15).

M. Rimer (LEM Dynamic Analysis, Control Dynamics Group, GAEC) for his work on the Polynomial Guidance Law, for generating a rigorous planar (3 degree of freedom) representation of the LEM dynamic and flight control system; and for his work on solving the stability problem exhibited between the Polynomial Guidance Law and the LEM vehicular representation (See Ref. 11, 16).

H. Sperling (LEM Dynamic Analysis, Trajectory Analysis Group, GAEC) for his invention and development of the novel Polynomial Guidance Law specifically for the hover to touchdown maneuver; and for his effort which contributed to solving the stability problem previously mentioned (See Ref. 11, 16).

The references for other work pertaining to this report are presented in the Reference Listing, at the end of the report.

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## 1.0 INTRODUCTION

The purpose of this study is to determine the impact on the LEM design resulting from a requirement for an automatic mission, with the LEM automated to a degree consistent with the ground rules listed below. In particular, the purpose of the study is to identify those changes in LEM equipment configurations and performance requirements that are necessary to provide an automatic mission capability.

All descent phases of the mission are examined briefly. However, because the automation of the nominally manual, hover to touchdown phase produces the most significant change in the LEM design, the study is essentially devoted to only this phase of the mission. The aspects of the LEM system which receive the most emphasis are those concerned with navigation, guidance, and flight control, including the crew participation in these operations.

## 2.0 GENERAL

### 2.1 GROUND RULES

(1) The automatic mission shall be considered to be a possible mode of operation for the basic manned lunar landing mission.

(2) The automatic function shall be achieved by utilizing the existing LEM configuration and flight plan where reasonable and practical. The change made will be modifications or additions to existing LEM equipment.

(3) The automatic LEM mission begins in lunar orbit prior to LEM-CSM separation and ends, for the purposes of this study, at touchdown on the lunar surface.

(4) Except for vehicle handling, the crew is allowed to perform functions that contribute to the operation of the LEM, such as:

a) manual IMU and backup guidance equipment alignment in orbit prior to descent.

b) manual insertion of initial conditions into guidance computer

c) monitoring of automatic Guidance & Control equipment operation.

d) manual switching for system activation and shutdown and override functions as required

### 2.2 AUTOMATION REQUIREMENTS FOR NOMINAL MISSION

The functions performed during the nominal mission are listed in Table I with their present means of implementation, either manual or automatic. Also shown in the table are the nominally manual functions which require automation according to the original ground rules for a completely automatic LEM, and the revised ground rules listed above.

The salient points that are demonstrated in Table I may be summarized as follows:

(1) Many of the functions required during the nominal mission are normally automatic.

(2) The revised ground rules of this study allow the crew to perform most of the manual functions which, if automatized, would require the addition of significant weight and complexity to the LEM system. In particular, utilization of the crew avoids the addition of:

- an automatic star tracker and possibly a star pattern recognition device for IMU alignment.
- equipment to provide for a remote surveillance of the landing area.
- equipment to implement the normally manual switching, systems activation and shutdown, data insertion, and monitoring functions.

Therefore, need for additional equipment under the ground rules which allow use of the crew will be negligible.

(3) The major problem area that remains is the automation of the hover to touchdown phase of the mission.

## 2.3 AUTOMATION OF HOVER TO TOUCHDOWN PHASE

### 2.3.1 General

Studies of the hover to touchdown operations involve investigation and development of suitable guidance laws, the evaluation of these laws with respect to satisfying the constraints of the problem and to being compatible with the flight control system, and the analysis of the sensitivity of these laws to guidance system component errors and initial conditions. To investigate the control system aspects, a simpler model of the FCS was included in the Proportional Navigation Law studies; and a more comprehensive model of the FCS is included in the Polynomial Guidance Law studies.

In the following sections, a typical guidance law that was studied for the hover to touchdown application is discussed in detail. This law is the Proportional Navigation Line of Sight Law. A modification to this law that improves its effectiveness for the hover to touchdown application is also discussed. The design and mechanization of the laws are discussed and their effectiveness in the hover to touchdown application is evaluated.

## TABLE I AUTOMATIC REQUIREMENTS BASED ON NOMINAL MISSION

X = additional automation functions required of LEM

- = additional automation functions not required of LEM

| MISSION PHASE | NOMINAL MISSION | | FUNCTIONS REQUIRING AUTOMATION | |
|---|---|---|---|---|
| | Functions | Means of Implementation | Complete Automatic Mission (Original Ground Rules) | Modified Automatic Mission Revised Ground Rules |
| Pre-separation (lunar orbit) | Take star sights, fine align IMU. | Manual | x | - |
| | Prepare N&G Computer. | Manual | x | - |
| | Activate, partially check radars and deactivate. | Manual | x | - |
| Separation | Translation for Separation. | Manual | - | - |
| | Check Rendezvous Radar (RR) tracking and skin track CSM with Landing Radar (LR), deactivate radars | Manual | x | - |
| Re-orientation for De-Orbit Boost (Injection) | Reorient vehicle prior to injection. | Automatic | - | - |
| Injection into Synchronous Transfer Orbit | Ignite descent engine and burn for required $\Delta V$. | Automatic | - | - |

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TABLE I AUTOMATIC REQUIREMENTS BASED ON NOMINAL MISSION (Cont'd)

| MISSION PHASE | NOMINAL MISSION | | FUNCTIONS REQUIRING AUTOMATION | |
|---|---|---|---|---|
| | Functions | Means of Implementation | Complete Automatic Mission (Original Ground Rules) | Modified Automatic Mission Revised Ground Rules |
| Descent Coast | RR activated, lock on and track CSM, deactivate. | Manual | x | - |
| | Sight stars, fine align IMU. | Manual | x | - |
| | RR activated, lock on and track CSM. | Manual | x | - |
| | LR self checks performed, deactivate both radars. | Manual | x | - |
| Powered Descent | Survey Landing site, monitor altitude. | Visual | x | - |
| | LR activated, final checked, remains on. | Manual | x | - |
| | LEM RR transponder activated. | Manual | x | - |
| | Vehicle attitude controlled for entire powered ascent. | Automatic | - | - |
| Hover to Touchdown | Survey landing point. | Visual | x | - |
| | Control the descent trajectory and attitude to touchdown. | Manual | x | x |
| | LR remains on. | Manual | x | - |

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2.3.2 Constraints

The following more important constraints have been selected (or dictated by LEM Design) for use in defining and obtaining acceptable hover to landing guidance law trajectories (also see Ref. 8).

(a) During Descent

1. Attitude and attitude rate limits -- The LEM thrust axis should be contained within ±30° of the local vertical. The maximum pitch attitude rates shall have peaks during descent of less than 10°/sec. However, during most of the descent, rates should be well below the peak limit.

2. Visibility -- Due to the fact that the vehicle will be manned, it is considered most desireable to have the landing sight and horizon in view for the complete hover to touchdown maneuver. However, loss of landing sight and horizon for short intervals of time is acceptable.

3. The required components of thrust in the inertial directions shall be obtained by orienting the main engine thrust vector via attitude changes.

4. Hover may start at altitudes as high as 1000 ft. with a zero vertical velocity and some horizontal velocity less than 100 ft/sec in the direction of the landing point.

(b) At Touchdown

1. Touchdown Velocities

   Horizontal - max. ±5 ft/sec
   Vertical - max. 10 ft/sec (down only)

2. Attitude rates at touchdown - max. ±5°/sec

3. $\Delta V$ used from hover to touchdown - Less than 650 ft/sec plus the horizontal velocity at hover.

4. Attitude at touchdown - max. ±5° off local vertical.

5. Touchdown miss distance - This value may or may not be critical depending on lunar terrain characteristics in the region of the landing site. Up to the present time, this value cannot realistically be determined. (However, it will be shown later that a great number of the trajectories using the proportional navigation law which miss their target by an appreciable amount also violate other touchdown criteria.)

### 2.3.3 General Trajectory Considerations

Although it would be desirable to achieve successful trajectories under all hover velocity and target ranges conditions, it should be recognized that this goal can not be realistically attained. This is basically due to the fuel (or $\Delta$V), thrust, and attitude limitations imposed. In many cases, the optimum vehicle attitude for visibility conflicts with the optimum for achieving the target point. For example, a high horizontal residual hover velocity may be useful in providing extended downrange distance but also results in great difficulty for landing within shorter ranges. To null this horizontal velocity rapidly would require high thrust levels coupled with large vehicle X axis deviations from the nominal vertical pitch attitude which would conflict with visibility and sometimes attitude rate restrictions. Limitations of approach velocity and target range values, together with judicious choice of guidance law gain constants has resulted in the generation of acceptable trajectory families for the application. This technique can in addition produce acceptable touchdown conditions and this is demonstrated by the work performed on the Proportional Navigation Law. It is also possible to cut off the law at some low altitude above the surface and replace it with a throttling back command for a vertical descent in an attitude hold mode until touchdown. This technique was applied to both the Polynomial and Proportional Guidance Laws with success.

The ability to keep the landing site within view is a function of the trajectory (altitude and range) and LEM attitude at each point. A means of improving the visibility to the landing point is to initially introduce a downward acceleration, which results in a "sagging" type of trajectory. Figure 1 shows two types of trajectories which may be obtained. "A" is obtained if a horizontal velocity is present at hover and "B" would be obtained if an initial downward acceleration were introduced in addition to the initial velocity.

Table 2 presents a comparison of some of the relative advantages and disadvantages of the two trajectories. It is considered that the "B" type trajectory is most advantageous especially with respect to the important visibility consideration. It is also possible to improve "B" to get some of the advantages of "A" by providing a cut off to the guidance law and introducing a vertical descent at sufficiently high altitudes above the lunar surface. In addition, as will be seen later in the report, trajectory "B" is obtained from an adaptive form of the Proportional Law so that the disadvantages of "B" for distant targets can be almost completely offset by commanding reduced downward accelerations.

TABLE 2 - TRAJECTORY COMPARISON

| | A | B |
|---|---|---|
| View of landing point | Lose view while still near maximum altitude. | Landing point in sight for essentially the whole maneuver. |
| View of ground before landing point | Ground is not within close view until final vertical descent is underway | Closer view of ground earlier in flight. |
| Change in landing site | Should be more capable for achievement of further downrange targets, since vehicle remains at higher altitude for most of trajectory. | Has an initial downward velocity which decreases the altitude early in the flight. This is favorable for achieving close in targets. |
| Landing on terrain which is peaked or rubble surfaced | Superior since final touchdown path is more vertical. | Tends to approach on a glide slope more suitable for landing on flat surface. |

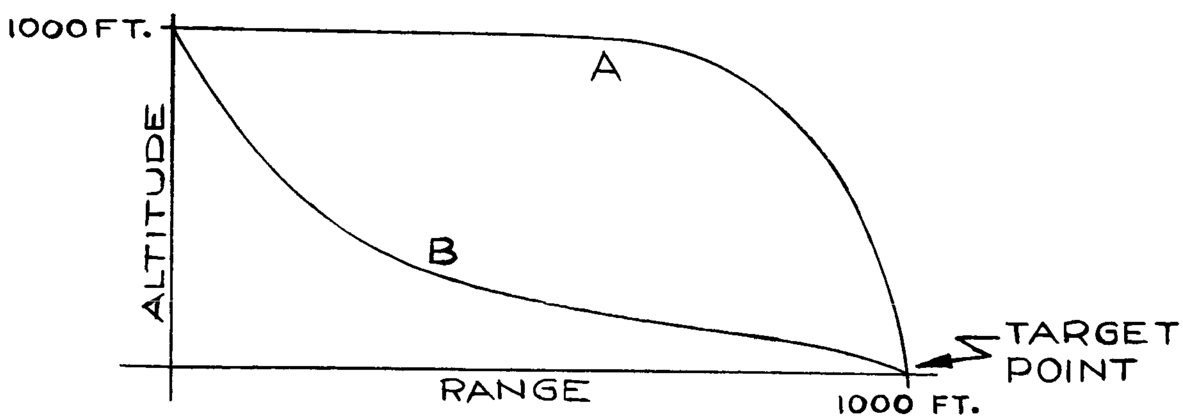


Figure 1 - Comparison of Trajectory Shapes

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## 3.0 BASIC & MODIFIED PROPORTIONAL NAVIGATION LINE-OF-SIGHT LAWS

### 3.1 INTRODUCTION

In this section, the GAEC study (Ref. 5, 6 and 7) which attempted to apply L. Cicolanis' Proportional Line of Sight Navigation Guidance Law (Ref. 4) to the hover to touchdown problem will be discussed. Although this law had been applied with success for other phases of the powered descent (Ref. 10), it had not previously been applied for the hover to touchdown maneuver. The discussion which follows presents the Basic Proportional LOS Law first and then Modified Proportional LOS Law (called MLOS) which appears to be better suited to the hover to touchdown phase (Ref. 5).

### 3.2 THEORY OF BASIC LOS LAW

Ref. (4) has derived and presented the guidance law for acceleration required with respect to an inertial frame for the Proportional Line of Sight Navigation Guidance Law. It is presented as equation (1) below.

$$\overline{f} = \overline{V} + 2\,\overline{W}_{Ia} \times \overline{V} + \overline{W}_{Ia} \times (\overline{W}_{Ia} \times \overline{R}) + (\overline{f}_t - \overline{f}_v) \qquad (1)$$

where

$\overline{f}$ = thrust acceleration vector of the vehicle, or thrust force per unit vehicle mass. This determines the required vehicle thrust magnitude and direction.

$\overline{f}_v$ = external acceleration vector or force per unit vehicle mass acting on the vehicle (gravity, aerodynamic, etc., as applicable). This force is defined positive along $\overline{V}$ and $\overline{W}_R \times \overline{V}$.

$\overline{f}_t$ = acceleration of the target $\frac{d^2\overline{R}_{Ia}}{dt^2}$ (with respect to an inertial frame).

$\overline{W}_R$ = vector angular velocity of line of sight between vehicle and target.

$\overline{W}_{Ia}$ = angular velocity of the reference frame with respect to an inertial frame.

$\overline{R}$ = vector LOS range between vehicle and target. Positive from target to vehicle.

$\overline{V}$ = relative velocity between vehicle and target ($\dot{\overline{R}}$), always positive since this is the reference.

$\dot{\overline{V}}$ = relative acceleration between vehicle and target ($\ddot{\overline{R}}$) which would be commanded by the guidance law if there were no target motion, or target coordinate axes rotation of the reference frame, or external vehicle forces.

Equation (1) may be simplified, since it is determined that there are no rotations of the reference frame relative to the inertially fixed coordinate system, or accelerations of the target point. The reference frame is centered at the target point. As a result $\overline{W}_{Ia} \times (\overline{W}_{Ia} \times \overline{R})$, $\overline{f}_t$, $\overline{W}_{Ia}$ are all zero, and (1) reduces to

$$\overline{f} = \dot{\overline{V}} - \overline{f}_V \qquad (2)$$

The expression for $\dot{\overline{V}}$ (the relative Proportional Guidance Law acceleration command) is derived in Ref. 4, and given below:

$$\dot{\overline{V}} = \frac{V^2}{R}\left[(S - 2) \text{ Cos } L - \left(S - \frac{K+1}{K}\right) \overline{u}_V + S\overline{W}_R \times \overline{V}\right] \qquad (3)$$

where (also see Fig. 1)

R = scalar range

V = scalar relative velocity between vehicle and target, Vo = initial value of V

S, K = are guidance law constants

L = the lead angle, i.e., angle between the relative velocity vector and the negative line of sight to the target point.

$\overline{W}_R$ = vector angular velocity of line of sight

$\gamma$ = flight path angles measured from velocity vector $\overline{V}$ to the local horizontal

The expression (3) is composed of two perpendicular vector components. One is along the velocity vector (See Fig. 2), direction $\overline{u}_V$, and the other ($S\overline{W}_R \times \overline{V}$) is perpendicular to the velocity vector. The expression (3) may be substituted in (2) to obtain the inertially referenced acceleration vector command or force per unit mass required.

$$\overline{f} = \frac{V^2}{R}\left[(s - 2) \cos L - \left(S - \frac{K+1}{K}\right) \frac{\text{Sin } L}{L} \overline{u}_V + S\overline{W}_R \times \overline{V} - \overline{f}_V\right] \qquad (4)$$

The only net external acceleration or force per unit mass acting on the vehicle is gravity and this acceleration may be resolved along the vector velocity and its perpendicular (in this case, planar flight) if the flight path angle ( $\gamma$ ) is known.

Assuming a configuration of velocity vector, local vertical, local horizontal, vehicle and target point as shown in Figure 2, it is seen that the gravity component will be in the direction of the $\overline{N}$ and $\overline{V}$ vectors. Therefore the gravity terms are substituted in (4) for $f_V$ with a positive sign, since $f_V$ is defined positive along $\overline{N}$ and $\overline{V}$. The result is that the gravity terms are subtracted since $f_V$ has a negative sign in (4) and the $\overline{f}$ thrust is as follows:

$$\overline{f} = \frac{V^2}{R}\left[(S - 2)\ \text{Cos}\ L - \left(S - \frac{K+1}{K}\right)\right]\overline{u}_v - g\ \sin\gamma \cdot (\overline{u}_v) \qquad (5)$$

$$+\ S\overline{W}_R \times \overline{V} - g\ \text{Cos}\ \gamma \cdot (\overline{u}_\phi \times \overline{u}_v)$$

where $\overline{u}_\phi$, $\overline{u}_V$ are unit vectors representing LOS rotational and velocity vector directions respectively.

The components of $\overline{f}$ may be listed as the acceleration along the velocity vector ($\dot{\overline{V}}_c$) and normal to the velocity vector ($\dot{\overline{N}}_c$). They are defined as follows:

$$\overline{f} = \dot{\overline{V}}_c + \dot{\overline{N}}_c$$

$$\text{where}\quad \dot{\overline{V}}_c = \frac{V^2}{R}\left[(S - 2)\ \text{Cos}\ L - \left(S - \frac{K+1}{K}\right)\frac{\text{Sin}\ L}{L}\right] - g\ \sin\gamma \qquad (6)$$

$$\dot{\overline{N}}_c = S \cdot \left|\overline{W}_R\right| \cdot \left|\overline{V}\right| \cdot\ \text{Sin}\ (\measuredangle\ \text{between}\ \overline{W}_R\ \text{and}\ \overline{V}) - g\ \cos\gamma$$

$\dot{\overline{N}}_c$ may be reduced further since $\left|\overline{W}_R\right| = \frac{V}{R}\ \text{Sin}\ L$

$$\dot{\overline{N}}_c = S\left(\frac{V}{R}\ \text{Sin}\ L\right) \cdot (V) \cdot \text{Sin}\ 90° - g\ \text{Cos}\ \gamma = S\frac{V^2}{R}\ \text{Sin}\ L - g\ \text{Cos}\gamma \qquad (7)$$

Note that the angle between $\overline{W}_R$ and $\overline{V}$ is 90°. Therefore the acceleration guidance law commands may be expressed as follows:

Along the velocity vector ($\overline{V}$)

$$\dot{\overline{V}}_c = \frac{V^2}{R}\left[(S - 2)\ \text{Cos}\ L - \left(S - \frac{K+1}{K}\right)\ \frac{\text{Sin}\ L}{L}\right] - g\ \sin\gamma$$

Normal to the velocity vector (in direction $\overline{u}_\phi \times \overline{u}_V$)

$$\dot{\overline{N}}_c = S\frac{V^2}{R}\ \text{Sin}\ L - g\ \text{Cos}\gamma$$

The commanded angle ($\alpha$) of the acceleration vector relative to the velocity vector $\bar{V}$ is specified by the equation

$$\alpha = \tan^{-1}\left(\frac{\dot{N}_c}{\dot{V}_c}\right) \qquad 7a$$

and the commanded angle with respect to the local horizontal is

$$\theta_c = \gamma + \alpha \qquad 7b$$

## 3.3 THEORY OF MODIFIED LOS LAW

L. Cicolanis's LOS Proportional equation (Ref. 4) is readily adaptable to automatic feedback system usage since it generates vehicle commands directly based on vehicle state data. It has been used for several analyses concerned with powered descent (See Ref. 10), and it yielded satisfactory results in that it produced near optimum $\Delta V$ trajectories. One obvious fault with the basic law (for the hover to touchdown phase of the flight) is that a literal hover condition results in no guidance command (except enough to offset the gravity forces, see equations (9) and (10). This could result in a continually commanded hover condition, which would be intolerable for this phase of the mission.

A. Bierman, in a GAEC memo, Ref. 5, presents a modification to the Cicolani's Proportional Navigation Law, which was developed to overcome the above difficulty. In order to preserve the basic nature of the unmodified Proportional Navigation Law, it was decided that only the external force portion (the gravity terms) would be modified and that phase-in of the modification would occur at start of hover and phase-out would occur as soon as vehicle velocity was brought up or down to a value acceptable to the basic guidance law. The acceptable value of desirable velocity is obtained from equation (10) once the time of flight is selected. The other quantities (L and R) are known and K is selected. Since the vehicle thrust-to-weight ratio ranges within narrow limits for this phase and the $\Delta V$ used is small, in relation to the $gI_{sp}$ product, $\Delta V$ increases nearly linearly with time. Therefore, controlling time, indirectly controls $\Delta V$ in a near linear fashion. The Modified Law therefore provides a strong influence on the control of $\Delta V$ independently of K. For this phase of the mission, when the remaining fuel is approaching the minimum, this is regarded as a desirable feature.

In addition the Modified Guidance Law contains an adaptive feature, as implied previously. The modification to the basic law tends to bring vehicle velocity conditions from a region which would present great difficulty to the Basic Law (in meeting various constraints specified in section 4.0) to values which are more easily handled.

The equations derived in Ref. (5) for the Modified Guidance law are presented as follows:

$$\dot{V}_c = \frac{V^2}{R}\left[(S - 2)\,\text{Cos}\,L - \left(S - \frac{K+1}{K}\right)\frac{\text{Sin}\,L}{L}\right] - G\,(V/V_R)\,\text{Sin}\,\gamma \qquad (8)$$

where: $$\dot{N}_c = \frac{SV^2}{R}\,\text{Sin}\,L + G\,(V/V_R)\ \ \text{Cos}\,\gamma \qquad (9)$$

$$G(V/V_R) = \left(\frac{V}{V_R}\right)^{y(t)}\cdot g$$

$V_R$ = velocity required

$$= V_R\,(t) = \frac{KR}{Tf}\ \frac{L}{\text{Sin}\,L} \qquad (10)$$

$T_v = T_{fo} - t$ = time of flight remaining from total pre-selected time ($T_{fo}$) after time elapsed, t.

$$y(t) = Y(t) = \frac{(t_2 - t)}{(t_2 - t_1)},\ t_1 \leqslant t \leqslant t_2 = \text{function} \qquad (11)$$

which describes a continuous transition from the G $(V/V_R)$ coefficient of Modified Guidance Law to the Basic Guidance Law coefficient g.

$t_1$ = the time at which the modified factor starts to be removed

$t_2$ = the time when the transition from Modified Guidance Law to Basic Guidance Law is completed.

Note that for t between 0 and $t_1$, y (t) = 1. (by definition). For t between $t_1$ and $t_2$, $y\,(t) = \frac{(t_2 - t)}{(t_2 - t_1)}$ and for $t \geqslant t_2$, y (t) = Y (t) = 0 (by definition).

The affect of the transition factor is to have the second terms of both (8) and (9) start out at beginning of hover as $\frac{V}{V_R}g$. At $t = t_1$, $\frac{V}{V_R}$ is forced to approach unity by a linearly varying function of time (11), in the exponential. Then at $t = t_2$ the second term equals g and remains there. At this point and thereafter, the Modified Guidance Law, (8) and (9), and the Basic Guidance Law (6) and (7), are identical. The remainder of the trajectory would be the same as that which would occur if the Basic Guidance Law were used, with the initial and boundary conditions which exist at the point of switchover. In general, these conditions would have been made more favorable for the Basic Law takeover.

Examination of the vector diagram in Fig. 2 and consideration of equations (8) and (9) and in particular the $\frac{V}{V_R}g$ term convey the adaptive feature of the Modified Guidance Law in further detail.

First, it is seen that for large values of V (those which exceed the required value ($V_R$) as determined in (10)), an acceleration in a direction opposite to that commanded by the guidance law is generated which exceeds the gravity compensation force normally generated under the Basic Law by the ratio $\frac{V}{V_R}$. The net result is that an upward thrusting force is commanded to the vehicle.

Second, it is seen that for small values of V (those which are less than required ($V_R$)), a net force which is less than that required to offset gravity is commanded. This results in an increase in the total velocity vector due to addition of a downward component of acceleration to the vehicle. This occurs until the velocity increases to the velocity required or until $t_1$, whichever occurs first. Due to vehicle attitude constraints, acceptable means of obtaining the increased downward velocity component is to reduce vehicle thrust to a minimum (the other alternative would be to invert the vehicle, which is clearly undesirable at this point). This value is limited to approximately .77 lunar g with minimum thrust of 1050 lbs. The result is a max. downward acceleration of 1.2 ft/sec$^2$ (vehicle mass of 366.23 slugs used).

An additional beneficial factor which results from the downward acceleration is improved visibility of the target landing point. This occurs since a downward acceleration tends to produce a sagging trajectory, which by the geometry of the trajectory and location of the target point reduces the magnitude of the LOS angle from the horizontal, and therefore increases the chances for an unobstructed view through the LEM windows. It should be noted that the Basic LOS law produces a bowed trajectory with a horizontal initial velocity. (See 2.3.3 or Ref. 5 for a discussion of trajectory shape desired.)

## 3.4 DIGITAL COMPUTER SIMULATION PROGRAM (See the Appendix for further detail)

### 3.4.1 Introduction

The Proportional Navigation Guidance Law was programmed on a digital computer for use in simulations of a lunar landing from hover to touchdown. The digital computer simulation included the LEM vehicle rotational and translational dynamics, equations of motion, and a math model of the LEM sensors and guidance system computational functions. An IBM 7094 digital program was used to simulate the above mentioned functions. The program was designed to simulate planar motions of the LEM vehicle with three degrees of freedom (two translational and one pitch rotational).

### 3.4.2 Program Content and Capabilities

The program contains the following characteristics and capabilities:

(1) A functional representation of an inertial platform with two orthogonally mounted integrating accelerometers lying in the plane of motion. The model permits the insertion of the following error sources: accelerometer bias errors, fixed platform drift rate, and initial platform misalignment.

(2) A functional representation of a two beam doppler radar whose outputs correspond to local vertical and horizontal measurements. The specific radar outputs are: altitude, altitude rate, and horizontal velocity. Provision is made to permit insertion of scale factor and bias errors in each of the three output channels, and to reflect the effects of platform misalignment.

(3) A throttleable main engine which thrusts along the LEM longitudinal axis. The engine is limited between an upper and lower thrust level, while the response is characterized by a simple first order time constant.

(4) Attitude control dynamics are represented by a linear second order system with rate and position feedback. This representation is rather simplified but adequate for purposes of the study. However, a more rigorous dynamic representation of the LEM vehicle was used in Polynominal Law study (Reference 16 and Section 4).

(5) A functional representation of a spacecraft digital computer which performs the navigation, guidance, and control computations, including the updating of target position and LEM velocity on the basis of radar and IMU information. Initial errors in position and velocity can be inserted into the navigation loop.

The following vehicle characteristics were used throughout the analysis:

| | | |
|---|---|---|
| Initial mass | = | 366.23 slugs |
| Initial pitch inertia | = | 9105.0 slug-feet$^2$ |
| Descent engine time constant | = | 0.3 seconds |
| Specific impulse of main engine fuel | = | 300.0 seconds |
| Computer sample rate | = | 2 cycles per second |

### 3.4.3 Functional Description of Program (See Fig. 3)

The guidance system consists of the IMU, doppler radar, and the spacecraft digital computer. The spacecraft computer calculates inertial velocity and position on the basis of IMU information. Simultaneously, the spacecraft computer, using the IMU as an attitude reference, calculates the local vertical and horizontal velocity components and the LEM altitude on the basis of the radar outputs. The navigation computer position and the doppler velocity components are then processed to yield updated inertial velocity components. The navigation computer position and the radar altitude measurement are used to update the target position. The updated information provides the inputs to the guidance law from where thrust and steering commands are issued. The system accuracy, independent of sensor errors, is dependent upon the computation rate. For a computation rate of 2 cps, the following navigational accuracies are attained after 120 seconds of real flight time: position error of 0.2 feet, velocity error less than $10^{-5}$ feet per second, error in predicted target altitude of 1 foot.

Figure 3 is a functional block diagram which describes the general computational flow of the digital program used.

The guidance computation (employing the Modified Proportional Navigation Law) accepts navigational inputs such as LEM velocity ($\bar{V}$) and position vector ($\bar{r} = r, \phi$). It also accepts target position vector ($\bar{r}_t = r_t, \phi_t$) inputs. From these inputs the guidance law commands a thrust magnitude command to the engine ($T_c$) and a LEM attitude pitch command ($\theta_c$) to the vehicle attitude control system (box entitled "Pitch attitude and engine dynamics"). The vehicle attitude control system and the engine respond to the commands received and the output are actual dynamic vehicle attitude and thrust achieved. The "actual" dynamic thrust magnitude ($T_a$) and direction ($\theta_c$) are applied to the LEM vehicle to produce translational motion in the lunar gravity field environment. The outputs of the translational dynamics box shown are actual LEM vehicle velocity ($\bar{V}_a$) and position vector ($\bar{r}_a$). Since translational dynamics result in vector acceleration, there is a change in the velocity vector ( $\Delta \bar{V}_{ta}$). It is possible to calculate this and use it for a simulated input to the IMU sensing box to represent vehicle acceleration. The IMU senses vehicle acceleration and reads out "measured" velocity change ( $\Delta X_a$, $\Delta Y_a$) to the navigation computational block. The navigational function integrates accelerations twice by summing velocity and position changes to produce LEM vehicle position vector ($\bar{r}_M = \phi_M, r_M$) and LEM velocity vector as measured. At this point the computation loop is closed by feeding velocity and position data to the Guidance law box. The IMU computation may accept measurement errors such as accelerometer bias errors, platform drift rate and initial platform misalignment.

The translation dynamics computational output of vehicle position vector ($\bar{r}_a$) is fed into the lunar surface simulation box. The input of surface slope and the altitude of the target surface point above the lunar mean radius allow computation of vehicle altitude ($h_a$) above the sloped lunar surface. This altitude serves as an input to the radar so that the radar sensor measurement may be simulated. Altitude range measurement errors also may be included. Based on the radar altitude measurement and previous navigation computations of X, Z (LEM position vector coordinates referred to the inertial reference systems) the value representing updated target radius is computed and transmitted to the guidance law computation box. The target angle input to the guidance law is pre-selected.

The translational dynamics output of vehicle velocity vector ($\bar{V}_a$) is also an input to the radar box so that computation of altitude rate measurement and horizontal velocity error will affect the computation of simulated radar measured values.

The total radar measured velocity values are fed into the update velocity computation where they are combined with navigational inputs of target position (X, Z). The updated velocity ($\dot{X}$, $\dot{Z}$) components of the vehicle are outputs which are fed to the navigation box for use in determination of the updated vehicle velocity vector ($\bar{V}$).

## 3.5 DISCUSSION OF BASIC LOS LAW

### 3.5.1 Theoretical Aspects of Gain Constants and Initial Conditions

#### 3.5.1.1 Introduction -

The Basic Proportional Line of Sight Navigation Law has two constants K and S (see eq. (6) and (7) in Section 3.2) which affect the characteristics of the vehicle's trajectory (shape, time of flight, $\Delta V$ used, attitude and attitude rates, velocity profile) and which, in turn, are sensitive to such initial conditions as vehicle velocity, vehicle attitude and target downrange distance. In the following sections, a theoretical discussion is presented on the limitations on K and S and their interactions with initial conditions for the Basic Proportional Law.

#### 3.5.1.2 Lower Limit on Constants -

Reference 4 states some of the theoretical limitations on the gain constants. They are again presented as follows: In order to avoid a singularity in the guidance law as both range and velocity approach zero, K must be selected so that it is equal to or greater than 2. See equations (6) and (7) in Section 3.2. Also S must be greater than 2/K. If K = 2, then S must be greater than 1. However the reference points out that peculiarities in the trajectories could occur for values of S less than 2. Even though successful trajectories could be obtained, S was restricted to values greater than 2. This would insure that the acceleration command term approach zero as range to go and velocity are reduced to zero. Thus to summarize, the values of K and S are constrained as follows:

$$\begin{aligned} K &\geq 2 \\ S &> 2 \end{aligned} \qquad (1)$$

The reference states that the lead angle, i.e., the angle between the velocity vector and the target line of sight (see Fig. 2), monotonically approaches zero as the vehicle approaches its target if

$$S > 1$$
$$K \geqq 1 \qquad (2)$$

Therefore, if (1) is satisfied, then the lead angle conditions resulting from (2) will also be satisfied.

3.5.1.3 Effect of Constants on Trajectory Shape and Time -

The shape of the trajectory is determined by S and the initial lead angle ($L_o$). The smaller the lead angle, the tighter (closer to the line of sight) the trajectory, which is to be expected. In addition the trajectory path gets tighter as S increases. It should be noted that S does not directly affect time of flight.

The time of the maneuver is affected by the value of K as may be seen by the formula:

$$t_f = K \frac{R_o}{V_o} \cdot \frac{L_o}{\sin L_o} \qquad (3)$$

But it should be noted that K does not affect the trajectory path shape.

Applying formula (11) from Section 3.5.1.5 $\frac{V_o^2}{R_o^2} < \frac{g}{Sh_o}$,

and (3) above, and the approximation $\frac{L}{\text{Sin}_L} \doteq 1$, the relationship between K and S for an initial altitude of 1000 ft. above the lunar surface, is found as follows:

$$t_f = \frac{K R_o}{V_o}$$

The $\Delta V$ constraint is met if $t_f < 120$ sec. Therefore the inequality

$$\frac{K R_o}{V_o} < 120$$
$$\frac{K}{120} < \frac{V_o}{R_o} \qquad (3a)$$

Substituting 3a in (11), it is determined that

$$\frac{K^2}{120^2} < \frac{g}{Sho}$$

$$K^2S < \frac{g(120)^2}{ho}$$

for $h_o$ = 1000 ft. and lunar g = 5.31

$$K^2S < 77 \qquad (3b)$$

The formula places a constraint on the $K^2S$ product based on $\Delta V$ from ( 3) and initial attitude limitations (from (11)). It has been found that greater K's (see discussion on Fig. 8c in Section 3.5.2.5) result in lower attitude rates at touchdown especially for high initial velocities. (Increase of K essentially prolongs flight time to allow the law more time to null higher velocities.) To get the largest K possible for nulling high velocities, (3b) is employed with the lowest S allowed. From (1) Section 3.5.1.2 $S > 2$. S = 2 substituted in (3b).

Therefore

$$K^2 < 38.5$$

$$K < 6.2 \qquad (3c)$$

If the attitude constraint is applied from (11) (See Section 3.5.1.5). Note that this attitude constraint is minimal and only prevents the vehicle from tilting back below the horizontal).

$$\frac{V_o^2}{R_o^2} < \frac{g}{Sho}$$

Then with values of S = 2.1, $h_o$ = 1000 ft., lunar g = 5.31, $R_o$ = 1440 ft.

$$V_o < \sqrt{\frac{g}{Sho}} \cdot R_o$$

$$< \sqrt{\frac{5.3}{2 \times 1000}} \cdot 1440$$

$$V_o < 74.2 \text{ ft/sec.} \qquad (3d)$$

Note that the minimum value of S allows the largest $V_o$ velocity to result in inequality (3d). This partially explains why $S = 2.1$ was selected for case 2 in Section 3.7.2; i.e., to extend the high velocity end attainable with the basic law without exceeding the attitude constraint.

To further discover the effect of K and S and initial conditions on other factors such as pitch attitude, attitude rates, fuel usage, and time of flight, GAEC (see Reference 2 and further Section 3.5.2 in this report), did some analytical and empirical investigation into the effects.

3.5.1.4 Initial Horizontal Velocity and Range Limitations Affecting the Time of Flight and $\Delta V$ Budget -

It has been empirically determined that the $\Delta V$ budget will not be exceeded if the time of flight is less than 120 seconds for the maneuver under consideration. Using this value of time, it can be shown (Reference 2 or from Equation 3) that if L is limited between 0+° and 90° and $K \geq 2$ (choose $K = 2$, the value which makes $V_{o_{min}}$ smallest), the minimum initial horizontal velocity to achieve a downrange target at $R_o$ slant range distance may be expressed as

$$V_{o_{min}} \doteq \frac{R_o}{60} \qquad (4)$$

for a 1000 ft. downrange target and an initial altitude of 1000 ft., $V_{o_{min}} = 23.6$ ft/sec. It may be seen from (3) that the minimum required horizontal velocity increases as the target downrange distance increases. The formula indicates the smallest initial velocity for which the target should be achieved by the vehicle within the 120 second time limit, which is concomitant with the $\Delta V$ budget. It should be noted that this minimum value of velocity does not necessarily insure meeting other constraints such as attitude, attitude rates or touchdown velocity. However, even though equation (4) does not insure meeting all constraints, it does give a minimum velocity bound (on a time basis) for the Basic Proportional Law, below which the desired trajectory cannot be achieved. The equation (4) does not apply to the Modified Proportional Law directly (presented in Section 3.3), but may be applied together with (3) to give insight into the relationship between initial velocity required and range. It may be stated in passing (refer to Section 3.3 for more detail) that the Modified Law must raise the vehicle velocity to a greater minimum value as indicated by application of (4) as the initial slant range distance ($R_o$) to the target is increased.

3.5.1.5 Initial Attitude Command Constraint Requirements Affected by Initial Velocity and Other Initial Conditions -

The angles $\gamma$ and $\alpha$ are defined as in Section 3.2, Figure 2 and text, and Figure 7 in this section. The angle commanded with respect to the local horizontal ($\theta_c$) depends on the direction of $\bar{V}$ and $\bar{N}$, since $\dot{\bar{N}}_c$ and $\dot{\bar{V}}_c$ are referenced from these quantities. The equation for $\theta_c$ is expressed as follows (see Equation 7a and 7b, Section 3.2 and Figure 1).

$$\theta_c = 360 - (\gamma + \alpha) \tag{5}$$

where

$$\alpha = \tan^{-1} \left( \frac{\dot{N}_c}{\dot{V}_c} \right) \tag{6}$$

If the vehicle has only a horizontal velocity (zero vertical velocity), then $\gamma = 0$ and

$$\theta_c = 360 - \alpha = 360 - \text{arc tan} \left( \frac{\dot{N}_c}{\dot{V}_c} \right) \tag{7}$$

If the guidance law equations, Section 3.2, Equation (6) and (7) are substituted in (7) above, it is seen that a fairly complex relationship exists in the determination of $\theta_c$ as follows.

$$\theta_c = 360 - \tan^{-1} \frac{-\frac{SV^2}{R} \sin L + g \cos \gamma}{\frac{V^2}{R} \left[ (S - 2) \cos L - \left(S - \frac{K+1}{K}\right) \frac{\sin L}{L} \right] - g \sin \gamma} \tag{8}$$

The largest pitch angle off vertical occurs at the initial portion of the trajectory. In order to determine an upper bound on initial velocity, as related to initial pitch angle, an upper bound initial pitch angle offset from the vertical is specified. This value is chosen to simplify the results of (8) as well as to constrain the vehicle attitude.

Inspection of (8) shows that the denominator $\dot{V}_c$ is negative for the ranges of constants under consideration. For the arc tan $\frac{\dot{N}_c}{\dot{V}_c}$ to result in the right $\theta_c$ command (between $90^\circ$ - $180^\circ$), it must fall in the range between $270^\circ$ - $180^\circ$ (see Eq. (7) ). This will be true if $\dot{N}_c < 0$.

3.5.1.5 (Cont.)

Therefore

$$g \cos \gamma - \frac{SV^2}{R} \sin L < 0 \qquad (10)$$

To constrain the angle between limits, such as $60^\circ$, and $120^\circ$, the right hand side of the inequality (10) could be obtained by substituting the angle in left hand side of (8) for the desired $\theta_c$ limits. However, this would require taking into account the denominator of (8) which affects the result quite radically since the bracketed quantity tends to be small and varies with changes in the constants and lead angle L. As a result it was decided to apply equation (10) to simplify the results. This would constrain the vehicle initial attitude to a maximum of $90^\circ$ back off the vertical.

Then upon substitution of $\sin L = \frac{h_o}{R_o}$ and $\gamma = 0$ in (10),

the expression becomes

$$\frac{SV_o^2}{R_o} \cdot \left(\frac{h_o}{R_o}\right) < g$$

or

$$V_o^2 < \frac{gh_o^2}{Sh_o} \qquad (11)$$

$$V_o < \sqrt{\frac{g}{Sh_o}} \quad R_o$$

This inequality (11) indicates an upper bound for initial horizontal velocity, provided the denominator of (8) is negative. Calculation for the values, lunar $g = 5.31$, $h_o = 1000$ ft. and $S = 3.1$ determines the inequality

$$V_o < .051 \; R_o$$

or

$$V_o < \frac{R_o}{20} \qquad (12)$$

If the preceding paragraph 3.5.1.4 is referred to it is seen that both an upper and lower bound for velocities is now obtained, based on initial maximum attitude and time of flight constraints as

$$\frac{R_o}{60} < V_o < \frac{R_o}{20} \qquad (13)$$

3.5.1.5 (Cont.)

Equation (11) indicates that the upper velocity bound tends to be reduced as S increases due to the fact that the attitude constraint may be exceeded. However, the effect is more complex for restricting pitch back off vertical to a specific value less than $90^{o}$ and the affect on the denominator of (8) must be studied for changes in S to get the total effect.

3.5.2 Computer Simulation Study of Gain Constant & Initial Condition Effects

3.5.2.1 General -

The LEM hover to touchdown phase was simulated by means of the IBM 7094 digital program described above. For the runs depicted in figures 8a, 8b and 8c, the following conditions were applied:

initial slant range ($R_o$) = 1414 ft.; initial altitude ($h_o$) = 1000 ft.; downrange distance ($d_o$) = 1000 ft.; guidance law gain constant S = 2.1. A discussion of the results follows.

3.5.2.2 Relationship Between Guidance Law Constants, Initial Velocity and $\Delta$V Required -

Figure 8a shows the relationship between K the guidance law constant, the vehicle initial horizontal approach velocity ($V_o$) at the hover point, and the $\Delta$V required to complete the hover to touchdown maneuver. It is seen that as K is increased for a given initial velocity that the $\Delta$V required increases in a near linear fashion. This is as predicted by equation (10) in Section 3.3 if the approximation of a linear relationship between $\Delta$V and time is assumed (in the same manner to that which is pointed out in Section 3.3). The Figure also shows that for a fixed K, the $\Delta$V required varies inversely with the initial velocity; also as would be expected from equation (10)and a $\Delta$V-time linear approximation.

Two of the boundary limit lines shows in the Figure (8a) are obtained from previous discussion; i.e. $K \geqq 2$ is determined from Section 3.5.1.2 and the upper $\Delta$V limit = 640 + $V_o$ is obtained from the constraints Section(2.3.2.b). The third limit line, which closes the cross hatched area in Figure 8a, corresponds to the attitude constraint, $\theta = 120^{o}$ max. The boundary line is obtained by picking off the corresponding values of K and $V_o$ from Figure 8b and plotting them on Figure 8a. The cross hatched area in Figure 8a therefore represents the total set of values of the guidance law gain constant K, as a function of initial velocities, that satisfies the pitch attitude and $\Delta$V constraints.

3.5.2.3 Relationship Between Guidance Law Constants, Initial Velocity and Resulting Vehicle Initial Pitch Angle -

Figure 8b shows that for lower velocities, the initial attitude angle is not greatly affected by K, which would be verified from an examination of equation (8) in Section 3.5.1.5. (Note that as K varies from 2 to 4, $\left(\frac{K+1}{K}\right)$ varies from 1.5 to 1.25 therefore affecting the value of tan $\theta$ by a very small amount in comparison to the changes in K).

If (8) is further examined, it is seen that the tan $\theta_c$ varies inversely as the square of the velocity. (g Sin$\gamma$ = 0). And since the tangent is very large for angles near 90° any reduction in the tangent magnitude by increase of elocity value will move the angle away from the 90° orientation as indicated in 3.5.1.5. In this case the angle moves away from the 90° vertical so that $\theta_i$ changes in an increasing direction. The Figure 8b bears out the conclusion that as the horizontal initial velocity increases the vehicle pitch back off the vertical increases.

3.5.2.4 Relationship Between Guidance Law Constants, Initial Velocity and Resulting Vehicle Pitch Rates at Touchdown -

Figure 8c is a plot showing the relationship between initial velocity and guidance law gain constant K as affecting the angular rate at touchdown. For all ranges of initial velocity (from 30-70 ft/sec) the landing limit of 5°/sec is not exceeded provided $K \geqq 2.5$. For the ranges of velocity of 30-55 ft/sec $K \geqq$ approximately 2.2. For k = 2 the velocity must be limited to below 40 ft/sec to insure an angular rate below the 5°/sec maximum. Figure 8a could reflect these limitations by removing the shaded portion along the $V_o$ = 40 ft/sec line to K = 2.2 and the K = 2.2 line from 40 ft/sec to approximately 55 ft/sec. As shown in Figure 8a, the piece formed by the dotted line would be removed.

3.5.2.5 Implementation -

The charts in Figures 8a, b and c suggest a possible implementation procedure which could be applied to allow successful landings to be completed using the (basic unmodified) Proportional Law. It would be possible to check initial vehicle velocity (using the radar or inertial system) and the K gain value against a chart of K, $V_o$ values (either manual or automatically). If the K and $V_o$ values are away from the optimum boundaries, the K value may be changed to obtain better trajectory results. For example, if high approach velocities exist, Figure 8c and Figure 8a suggest a good tradeoff. Assume an approach velocity of 60 ft/sec. with a K = 2.5; therefore

3.5.2.5 (Cont.)

$\Delta V$ = 280 ft/sec, which is well below the allotted amount. The angular rate at touchdown is $2.5^\circ$/sec. If it is desired to reduce the touchdown angular rate, it can be done at the expense of $\Delta V$ by increasing K to 3.5. This will result in a touchdown rate of approximately $.5^\circ$/sec while increasing the $\Delta V$ to 475 ft/sec, still well within the $\Delta V$ budget. Figure 8b shows that the initial pitch angle also increases by $8^\circ$ to $138^\circ$, so that the landing site is lost initially for a short time. However this situation is rapidly corrected and a much better terminal trajectory is obtained as a result.

At the opposite extreme of low approach velocities another favorable tradeoff may be transacted. For K = 3.0 and $V_o$ = 30 ft/sec Figure 8a indicates that the $\Delta V$ budget would be exceeded. If K is reduced to 2.0, however, the trajectory is within the $\Delta V$ budget. This is accomplished at an increase in touchdown angular rates as may be seen in Figure 8c from nearly $0^\circ$/sec to $2^\circ$/sec. If the $\Delta V$ remaining were considered an obsolute maximum, it would be incomparably better to touchdown with a higher angular velocity (still well within constraint of $5^\circ$/sec) than to run out of fuel while at some distance from the lunar surface.

The figures do not show other important final conditions such as horizontal and vertical velocity at touchdown, attitude at touchdown and miss distance. These were not included since they are well within the limit conditions. (Sample values are given in Table 1.)

The same type of runs, may be performed with S, K and $V_o$ and even $R_o$ and $h_o$, as variables, to examine possible tradeoffs obtainable for changes in S and K. When completed, the information could be stored in some manner and used to obtain the best trajectory available with the prevailing initial conditions that the vehicle enjoys (or does not enjoy). The application of this information would add an adaptive feature to the guidance law and result in improved trajectories for many cases.

3.5.2.6 <u>Results Obtained for Varying Target Downrange Distances</u> -

The previous data presented was limited to the application of the Basic Proportional Law at downrange distances of 1000 feet. Table 3 shows the results of some sample trajectories with varying downrange distance (constant 1000 ft. initial altitude) and varying initial velocities both for the basic LOS law (LOS stands for line of sight representing the Basic Law). Note that the ratio of $\frac{R_o}{V_o}$ is held constant for all runs at approximately

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TABLE 3: SOME HOVER-TO-TOUCHDOWN TRAJECTORIES FOR INITIAL RANGES-TO-GO GREATER THAN 1000 FEET

| | INITIAL CONDITIONS | | | | | FINAL CONDITIONS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | do (ft) | $VH_o$ (ft/sec) | K | S | $\theta_o$ (deg) | $\lvert\dot{\theta}\rvert_{MAX}$ (deg/sec) | $\beta_{AVG}$ (deg) | $\dot{h}_f$ (ft/sec) | $VH_f$ (ft/sec) | $\theta_f$ (deg) | $\dot{\theta}_f$ (deg/sec) | $\Delta V$ (ft/sec) | Miss Dist. (ft) | |
| L | 2000 | 50 | 2.5 | 2.1 | 98.7 | 8.2 | -47.7 | -0.6 | 0.5 | 91.1 | -0.3 | 579 | 2.0 | L |
| O | 3000 | 60 | 2.0 | 2.1 | 97.0 | 1.7 | -35.0 | -1.1 | 1.7 | 94.6 | 1.0 | 552 | 3.0 | O |
| S | 4000 | 80 | 2.0 | 2.1 | 99.7 | 1.3 | -30.7 | -1.2 | 2.4 | 96.9 | 1.1 | 539 | 4.0 | S |
| M | 2000 | 40 | 2.0 | 2.1 | 94.1 | 3.3 | -49.3 | -0.2 | 0.2 | 91.6 | -0.9 | 635 | 0.0 | M |
| | 2000 | 40 | 2.5 | 2.1 | 96.4 | 1.5 | -31.3 | -0.1 | 0.1 | 90.7 | -0.6 | 628 | 0.2 | |
| L | 2000 | 60 | 2.5 | 2.1 | 100.5 | 1.5 | -72.1 | -0.1 | 0.0 | 90.1 | 0.0 | 635 | 0.0 | L |
| | 2000 | 60 | 3.5 | 2.1 | 109.3 | 2.7 | -40.4 | 0.0 | 0.1 | 90.2 | -0.1 | 606 | 0.2 | |
| O | 2000 | 60 | 4.0 | 2.1 | 114.2 | 4.7 | -31.9 | 0.0 | 0.2 | 90.4 | -0.3 | 563 | 0.8 | O |
| | 2000 | 80 | 3.5 | 2.1 | 119.0 | 3.0 | -65.8 | 0.0 | 0.0 | 90.1 | 0.0 | 618 | 0.0 | |
| S | 2000 | 80 | 4.0 | 2.1 | 126.6 | 7.1 | -55.2 | 0.0 | 0.0 | 90.1 | 0.0 | 597 | 0.1 | S |

LEGEND:

$d_o$ — initial surface range from LEM to touchdown site

$VH_o$, $VH_f$ — horizontal velocity, initial and final values, respectively

K, S — guidance law parameters

$\theta_o$, $\theta_f$ — pitch attitude with respect to local horizontal, initial and final values, respectively

$\lvert\dot{\theta}\rvert_{max}$ — maximum pitch rate (magnitude) during flight

$\beta_{AVG}$ — average visibility angle of touchdown site measured from LEM Z-axis; $\beta_{AVG}\ \frac{1}{T_f}\int_0^{T_f}\beta\,dt$
where $T_f$ = total time of flight

$\dot{h}_f$ — final vertical velocity

$\dot{\theta}_f$ — pitch rate at touchdown

$\Delta V$ — characteristic velocity

Reproduced from Ref. 2

3.5.2.6 (Continued)

50 to 1 and satisfies the requirements of (13) Section 3.5.1.5 (although it is near the limit of the left hand side). It also may be seen the touchdown conditions are well met except for the one parameter in the case of $d_o$ = 4000 ft. (attitude at touchdown exceeding the constraint limit by 1.9°). At this point it should be noted that if the $R_o/V_o$ ratio were not within the bounds of (13) required touchdown conditions would not be well satisfied. Although no runs have been performed for varying downrange distances with the Modified Law it is expected that there will be no improvement in the attainment of greater downrange distances if initial velocities are low. This is due to the fact that, there is essentially an additional downward gravitational component command initially introduced by the Modified Law to build up vehicle velocity as discussed in Section 3.3.

For close in and moderate ranges and high velocities, the Modified Law is capable of showing a greater improvement over the Basic Law. This would also be the case for low velocities and moderate ranges.

The second half of Table 3 (labeled MLOS standing for Modified Line of Sight or Modified Basic Law) shows data for moderate ranges and velocities varying from 40 to 80 ft/sec. In addition K is varied and S is held constant. It is seen from examination of the quantities listed that acceptable trajectories are obtained for all values of K. It is expected that plots similar to that shown in Figure 8 for the Basic and Modified Laws for 2000 feet downrange distance would show more values outside the constraint limits for the Basic Law than for the Modified Law. This is due to the inherent adaptive feature of the Modified law as pointed out in Section 3.3. A more detailed discussion on the results of application of the Modified Line of Sight Guidance Law is presented in the next section.

## 3.6 MODIFIED LOS GAIN CONSTANTS & INITIAL CONDITION EFFECTS

### 3.6.1 Introduction

As indicated previously, the Basic Line of Sight Proportional Law is characterized by a minimum critical velocity below which the vehicle cannot reach the target in the specified time, and therefore, the allotted $\Delta V$ budget is exceeded. The extreme case would occur with a perfect hover or zero velocity vector at the hover point, where the guidance law would not issue any commands (except to offset gravity). This resulted in the development of a modification to the Basic Law as described in Section 3.3. A summary and digestion of the results of computer

3.6.1 (Continued)

runs which apply to the Modified Law is presented. In the cases which follow, the gain constant K and the initial velocity $V_o$ are varied. The affect on miss distance, $\Delta V$, initial pitch attitude, and attitude at touchdown are presented for downrange distances ($d_o$) of 1000 and 2000 ft. The S gain constant is held at one value (S = 2.1) for all runs.

3.6.2 Miss Distance as Affected by K Gain and Initial Velocity

Figures 12 and 16 illustrate the effect of changing K and $V_o$ on touchdown miss distance from the target point. As previously pointed out in the constraints, Section 2.3.2, the requirements for miss distance are not well defined, so that no absolute pass-fail criterion can be specified. However, study of the figures will show excesses and trends in the effects of K, $V_o$ variations on miss distances. Figure 12 is for a 1000 ft. target downrange distance and Figure 16 is for a 2000 ft. target downrange distance. For the 1000 ft. distance, if $V_o$ is held between 20-100 ft./sec. and K is held between 2.5 and 4.0, miss distances are very small (near zero in most cases, one max. case at 15 ft.). For the 2000 ft. distance if $V_o$ is held between 60 (perhaps lower) and 100 ft/sec. and K is held between 2.5 and 4.0, the miss distances are very small. The above comparison reflects the need for additional horizontal velocity as the downrange distance is increased. If K is held to between 3 and 2.5, then the velocity range can be expanded from 40-100 ft/sec. These results show a fairly large region of K and $V_o$ values for successful operation. However, they do suggest the advisability of biasing the approach velocity on the high side since high velocities (up to 100 ft/sec. or possibly better) are handled very well from the standpoint of attaining the target point with a minimum miss distance. It should be pointed out, however, that there are several high velocity trajectories for the Modified Basic Proportional Law where although the initial tilt back angle is kept reasonably small and overall visibility is initially improved, the vehicle overshoots the target but reverses itself while in flight and makes perfectly acceptable landings without missing the intended target point. On the other hand, the Basic LOS never causes vehicle overshoots while in most cases making acceptable landings (depending more on K, $V_o$, for successful landings). However, it has the disadvantage of providing no visibility to the intended landing point. Therefore, there is a tradeoff involved, in comparing the Modified Law to the Basic Law, between some vehicle overshoot with fair visibility and no vehicle overshoot but practically no visibility. It should also be indicated that if the Modified Law is applied, a fixed value of S greater than 2.1 may be used. As indicated in Section 3.5.1.3 values of S greater than 2.1 cannot be allowed for the Basic Law due to the initial attitude constraint. The flexibility of the choice in S for the

3.6.2 (Continued)

Modified Law will result in obtaining satisfactory trajectories with a wider range of K, $V_O$ values. Thus, perfect landings with near zero miss distances, which would be an improvement over the miss distances shown in Figure 12 and 16, will result.

3.6.3 $\Delta$V Required as Affected by K Gain and Initial Velocity

Figures 13 and 17 are plots of the $\Delta$V required for downrange distances ($d_O$) of 1000 and 2000 ft. respectively. They both show that the $\Delta$V limitation of 640 + $V_O$ is not exceeded for ranges of velocity of 20-100 ft/sec. and all values of K considered (2.0-4.0). In addition, for 1000 ft. downrange $\Delta$V is not exceeded over a velocity range of 0-100 ft./sec. The results for the Basic Law shown in Figure 8a Section 3.5.2 with regard to $\Delta$V budget indicate that for combinations such as K = 3.5, $V_O$ = 40, or K = 3.0, $V_O$ = 30, the budget is exceeded (as determined by formula (3), (4) in Sections 3.5.1.3, 3.5.1.4). For the Modified Law (Figure 13) at 1000 ft. downrange, these points do not exceed the $\Delta$V budget. In addition, a check of Figure 12 shows that miss distance is negligible. However, further inspection of Figure 16 for 2000 ft. downrange using the Modified Law shows a considerable miss distance of approximately 250 feet for K = 3.5, $V_O$ = 40 and would show a similar magnitude of miss distance for K = 3.0, $V_O$ = 30 ft/sec. The results for 2000 ft. downrange for the Basic Law were not obtained, however it is expected that the $\Delta$V budget would be exceeded by an even greater amount than for the 1000 ft. case. So that although the improvement over the Basic Law by the Modified Law for $d_O$ = 2000 ft. is questionable, the improvement is considerable for the case of 1000 ft. downrange distance.

Reducing the value of K for low values of $V_O$ will help the Modified Law just as it helped the Basic Law to meet the constraint requirement, as discussed in Section 3.5.2.5 However, it is expected that fewer adjustments of K would be required for the Modified Law.

3.6.4 Initial Pitch Angle as Affected by K Gain and Initial Velocity

Figures 14 and 18 for 1000 and 2000 ft. respective downrange distances show the vehicle initial total pitch back angle measured between the horizontal and the vehicle positive X axis, when the Modified Law is applied. For K values between 2.0 and 3.5 and 1000 ft. downrange to target, the design constraint angle of 120° is not exceeded for velocities up to 60 ft./sec. The velocities can go up to 80 ft/sec. for 2000 ft. downrange to target. However, Figure 8b illustrating pitch angle when the Basic Law is applied shows that the design constraint is exceeded for the same K range with lower velocities at approximately 53 ft/sec. It also appears that the pitch back angle increases very rapidly in the Basic Law and much less rapidly in the Modified Law as velocity is increased above 60 ft./sec. This occurs in the Modified Law because a large contribution of the total thrust vector, command comes from

3.6.4 (Continued)

amplification of the gravity term. This increases the upward component of thrust and in turn makes the total thrust command closer to vertical, which the vehicle attitude follows (a rigid connection between x axis and engine had been assumed for the evaluation of the Proportional Law). In conclusion then, there appears to be a decided advantage in the Modified Laws' ability to take larger velocities without requiring as great an initial vehicle pitch back angle as needed for the Basic Law.

3.6.5 Touchdown Pitch Angle Rates as a Function of K Gain and Initial Velocity

Table 4 illustrates the affect of initial velocity and K gain constant on touchdown angular rates of a 1000 ft. downrange target. Examination of the table indicates that 5°/sec. is exceeded (by 3.1°) in only one case for K = 2 and $V_O$ = 20 ft./sec. All other combinations of K and $V_O$ give satisfactory touchdown attitude rate. As more time is allowed via increase in K, it is seen that the touchdown angular velocity is reduced to very small values. Comparison of Table 4 with Figure 8c shows that although the Basic (unmodified) Law is adequate for values of K above approximately 2.2, the Modified Law results in touchdown angular rates below 5°/sec. over a greater spread of K and $V_O$ values (including most K = 2.0, $V_O$ combinations).

3.6.6 Pitch Attitude at Touchdown as a Function of K Gains and Initial Velocity

Figures 15 and 19 show the pitch attitude at touchdown for a target at 1000 and 2000 ft. downrange. Figure 15 indicates that the vehicle touchdown attitude at 1000 ft. downrange is satisfactory for all but one of the K, $V_O$ combinations (K = 4, $V_O$ = 0) and this is only past the 5° bound by 1°. A small initial velocity would probably put it within the limit. Table 5 shows pitch attitude at touchdown using the Basic Law. If a comparison is made between the Modified and Basic Laws for various K, $V_O$ combinations which are common (K = 4 is not included in Table 3, so no comparison can be made for this value), it is seen that the Basic Law produces almost equally satisfactory results with the exception of the K = 2, $V_O$ = 70 point where touchdown attitude exceeds the limit 5° by 1.5°. Figure 19 showing pitch attitude at touchdown for 2000 ft. range, does not show results as good as that obtained for 1000 ft. downrange (both using the Modified LOS Law). Values of velocity must exceed 40 ft/sec. in order to result in good attitude touchdowns or else K must be lowered below 3.5 if the velocity exceeds 20 ft/sec. This indicates that greater velocities are required to achieve greater downrange landing points, if a change in K is to be avoided. On the other hand, if changes in K are allowed, then the system can perform satisfactory landings at greater downrange distances.

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TABLE 4

Pitch Attitude Rates ($\dot{\theta}_f$), (°/sec.) at Touchdown - Modified LOS Law, 1000 ft. Downrange Distance

| K \ $V_o$ ft/sec. | 0 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|
| 2 | .1 | 8.1 | 2.0 | .5 | 2.2 | 2.9 |
| 2.5 | .3 | 1.0 | .3 | .5 | .5 | .3 |
| 3.0 | .3 | .0 | .0 | .4 | .1 | .1 |
| 3.5 | .2 | .1 | .0 | .1 | .0 | .0 |
| 4.0 | .4 | .2 | .0 | .0 | .0 | .0 |

TABLE 5

Pitch Attitude ($\theta_f$), (degrees) at Touchdown - Basic LOS Law, 1000 Downrange Distance

| K \ $V_o$ ft/sec. | 28 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 75 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 90.4 | | 91.1 | 90.5 | 91.3 | 92.2 | | 96.5 | 93.4 |
| 2.5 | | 90.0 | | 90.2 | | 90.3 | 90.3 | 89.2 | |
| 3.0 | | 90.0 | | 90.0 | | 90.0 | 90.0 | 90.2 | |
| 3.5 | | 90.0 | | 90.0 | | 90.0 | 90.1 | 90.1 | |

3.7 COMPARISON OF MLOS AND BASIC LOS LAWS FOR SPECIFIC CONDITIONS

3.7.1 Case 1: Zero Initial Velocity at Hover, Vehicle at 1000 ft. Altitude, Target 1000 ft. Downrange ($d_o$)

Figures 9a - e show the more important characteristics of a trajectory produced from application of the Modified Law. It should be pointed out that the trajectory is obtained with a zero initial velocity at the hover point and that the Basic Guidance Law could never achieve the target point for reasons mentioned in Section 3.3. Therefore, no detailed comparison can be made between the two except that the Basic Law is clearly unacceptable for the perfect hover application. Examination of the figures shows that the vehicle performs the trajectory and achieves the target point meeting all constraints, as stated in Section 2.3.2, when the Modified Law is used. It does so without losing window visibility to the target, and has the horizon in view for practically all of the flight. Pitch attitudes and pitch rates never approach the maximums allowed and they approach 90° and zero °/sec respectively as the target point is approached. Thrust levels are reasonably constant after the initial dip, and vertical velocity is less than 10 ft/sec with approximately 470 feet downrange to go and 300 ft. attitude. Horizontal velocity is less than 5 ft/sec with 70 feet to go downrange and the vehicle altitude at approximately 35 feet. Touchdown velocity and angular rate conditions are much less and approximately approach zero.

Whether the target and horizon are within visibility limits is determined by the magnitude of the angle defined in Figure 9b. This is the angle ($\xi$) between the vehicle Z axis and the line of sight to the horizon. Due to LEM window configuration the horizon is considered visible when $10° \geq \xi \geq -65°$ measured from the Z axis. In addition, the target is visible for $10° \geqq \beta \geqq -65°$, where $\beta$ is the angle between the vehicle Z axis and the line of sight to target.

3.7.2 Case 2: 40 ft/sec Initial Horizontal Velocity at Hover, Vehicle at 1000 Ft. Altitude, Target 1000 Ft. Downrange ($d_o$)

Figures 10 a - e describe the trajectory produced by application of the Modified Law for the above conditions. Figures 11a - e describe the trajectory produced by application of the Basic Law for the same conditions. The constants chosen for both laws are identical except for the value of S used. For the Basic Law S = 2.1 is chosen. For the Modified Law S = 4.0 is chosen. The choice of a fixed S is determined from the standpoint of giving the largest number of acceptable trajectories with widest variations of K and $V_O$. See Section 3.5.1.2, 3.5.1.3 which discusses the limitations on S and see Figure 8, Section 3.5.2 on K, $V_O$ variations for the Basic Law.

Figure 10a and 11a - Vertical velocity starts to decrease from its maximum point of 19 ft/sec. much earlier using the

3.7.2 (Continued)

Modified Law (Figure 10a) than with the Basic Law (Figure 11a), (Range to go of 525 vs. 300 ft.). Horizontal velocity is below the touchdown limits of 5 ft/sec. with 100 ft range to go to target in Modified case, vs. 75 ft. to go in the Basic case. Total velocity is down much more rapidly in the Modified case as the landing point is approached.

Figure 10b and 11b The horizon (determined by $\xi$ ) is within sight for both cases. The target is within sight (Just barely) for most of the descent but gets better as the target is approached for the Modified case. For the Basic case, the target (determined by point B) starts in sight but is lost and gets worse as the target downrange distance closes to 500 ft., a considerable distance from the landing site. It should be noted here that this is one of the best visibility trajectories for the Basic Law (for $d_o$ = 1000 ft.) and in general results have been that improvement in visibility have been more decisive in favor of the Modified Basic Law.

Figure 10 c and 11c - The Basic Law exhibits higher altitude characteristics for the same downrange distance. This may be of some advantage depending upon the severity of terrain conditions. If this problem for the Modified trajectory, a fixed altitude bias may be inserted and the touchdown may be completed with a vertical letdown.

Figures 10d and 11d - Vehicle Pitch Attitude are within the attitude constraint limits, but the Modified trajectory produces an upright vehicle position much sooner than the basic, although the initial vehicle lean back is somewhat greater for the Modified case.

Vehicle pitch rates in the Modified case hit a peak of 7.5°/sec at the start of the trajectory whereas pitch rates in the Basic case are less than 1°/sec for the whole maneuver. Pitch rates however get below the allotted 5°/sec rate for touchdown before 30% of the time to go has elapsed, and at touchdown approach zero.

Figures 10e and 11e- Thrust profiles are good for both with no abrupt changes required. The $\Delta$ V's for both trajectories are approximately the same.

Conclusions - Because of its ability to complete a trajectory starting at a zero velocity hover and due to lower landing approach velocities and superior visibility, it appears that the Modified Law would be preferred.

Comparison of the Modified and Basic Guidance Laws for cases of velocity between 0 - 40 ft/sec will yield much the same results

(Continued on P. 82)

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DETERMINATION OF DIRECTION OF $\overline{N}$ (REF 4)

1. THE DIRECTION OF THE ROTATION VECTOR $\overline{\omega}_R$ IS DETERMINED BY $\overline{\mu}\phi$. ($\overline{\omega}_R = \Omega_R \overline{\mu}\phi$)
2. DIRECTION OF $\overline{\mu}\phi = \overline{\mu}_R \times \overline{\mu}_V$.
3. SINCE $\overline{N} = \overline{\omega}_R \times \overline{V}$, DIRECTION OF $\overline{N}$ IS DETERMINED BY $\overline{\mu}\phi \times \overline{V}$
4. IN DIAGRAM BELOW $\overline{\mu}_R$ IS POSITIVE IN ab DIRECTION, THEREFORE $\overline{\mu}\phi$ IS INTO THE PAPER AT POINT b & FINALLY $\overline{N}$ IS AS SHOWN.
5. $\overline{f}$ POSITIVE IS ALONG $\overline{V}$ & $\overline{N}$ DIRECTIONS

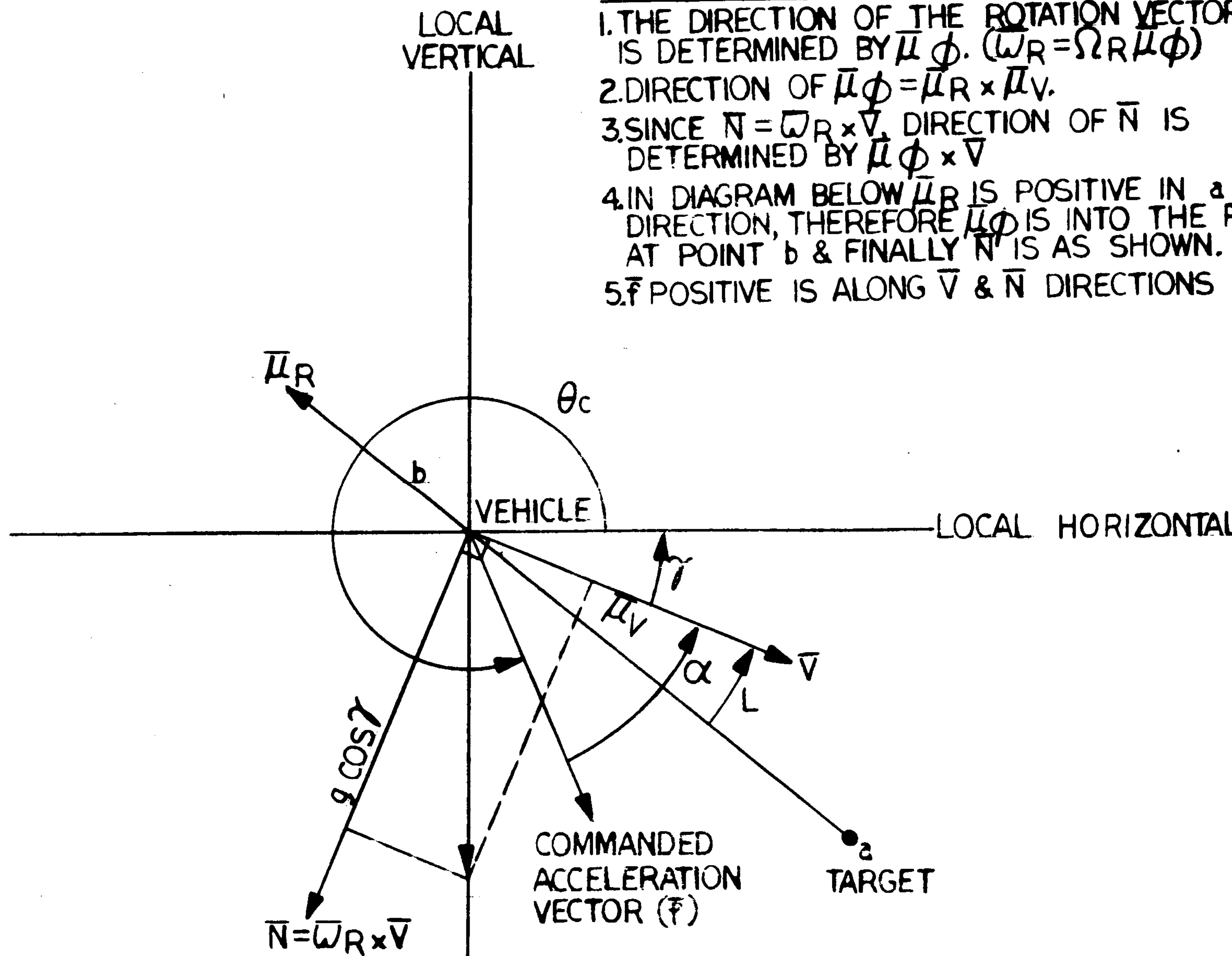


FIG 2

VECTOR DIAGRAM REPRESENTING DIRECTIONS FOR PROPORTIONAL LINE OF SIGHT GUIDANCE NAVIGATION LAW

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FIG 3

FUNCTIONAL DIAGRAM OF GUIDANCE SYSTEM USED FOR IBM 7094 DIGITAL ANALYSIS.

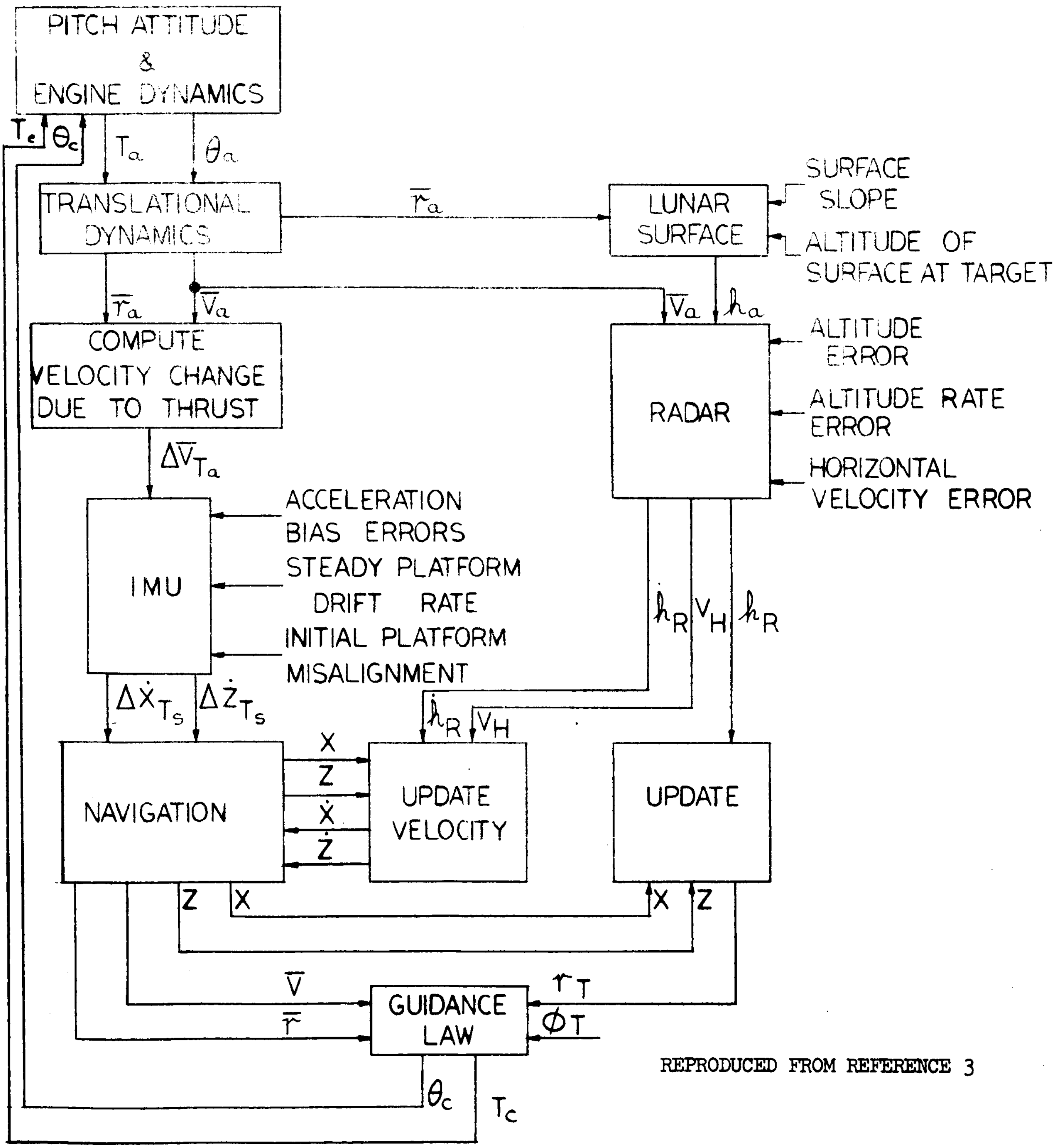


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FIG. 4

# COORDINATES FOR PLANAR TRAJECTORY

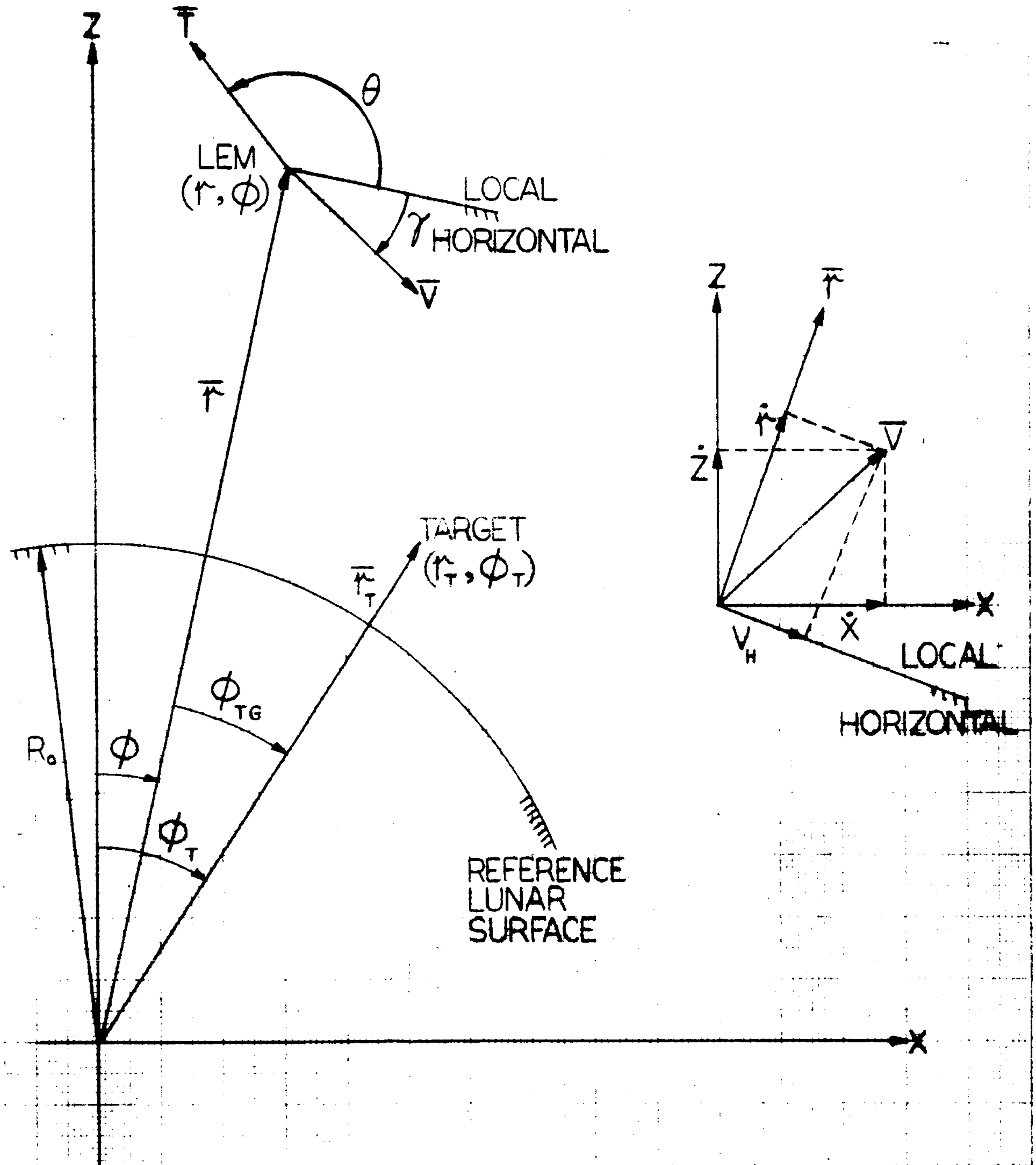


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FIG. 5

# GEOMETRY FOR SIMULATED INCLINED SURFACE

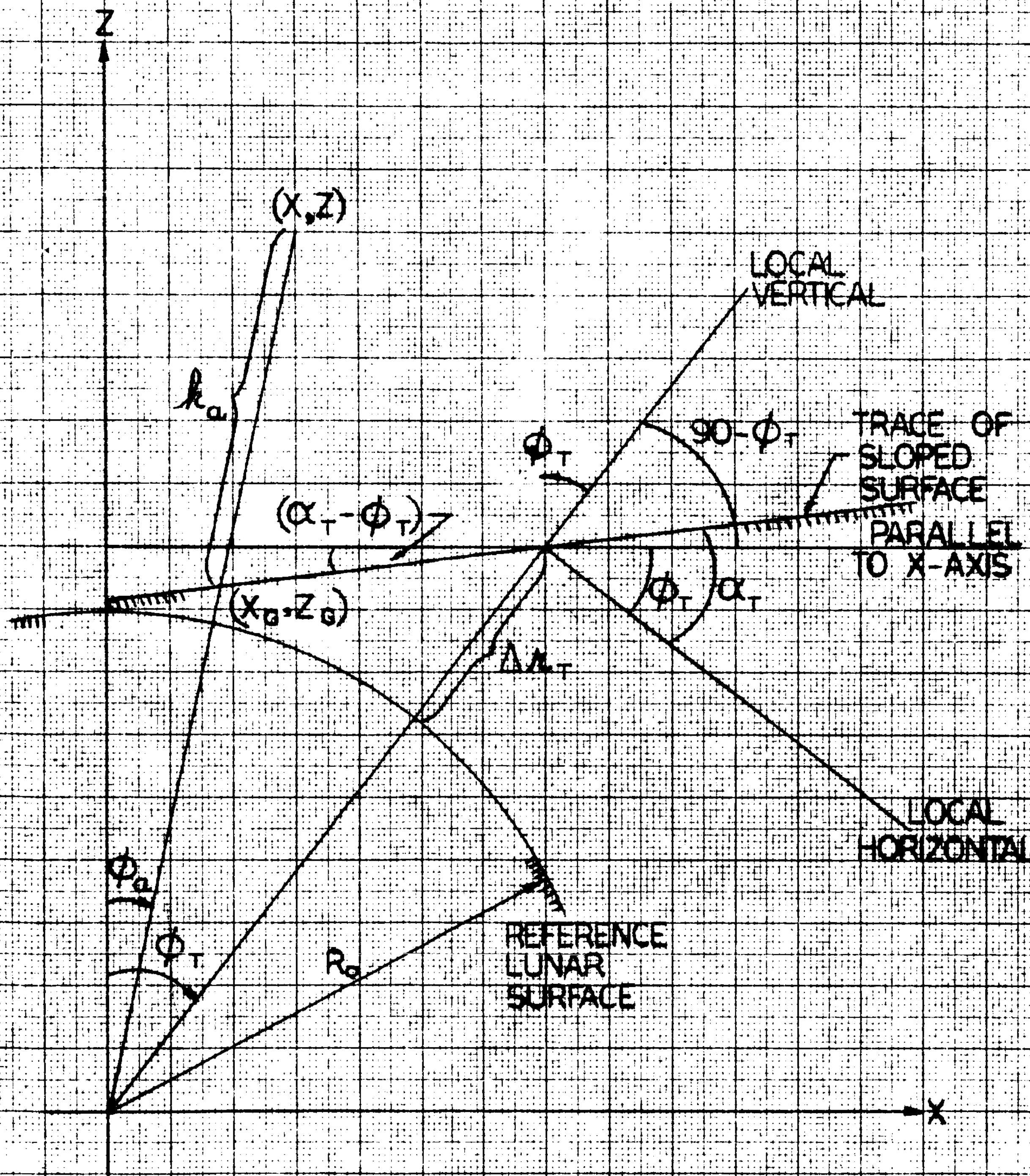


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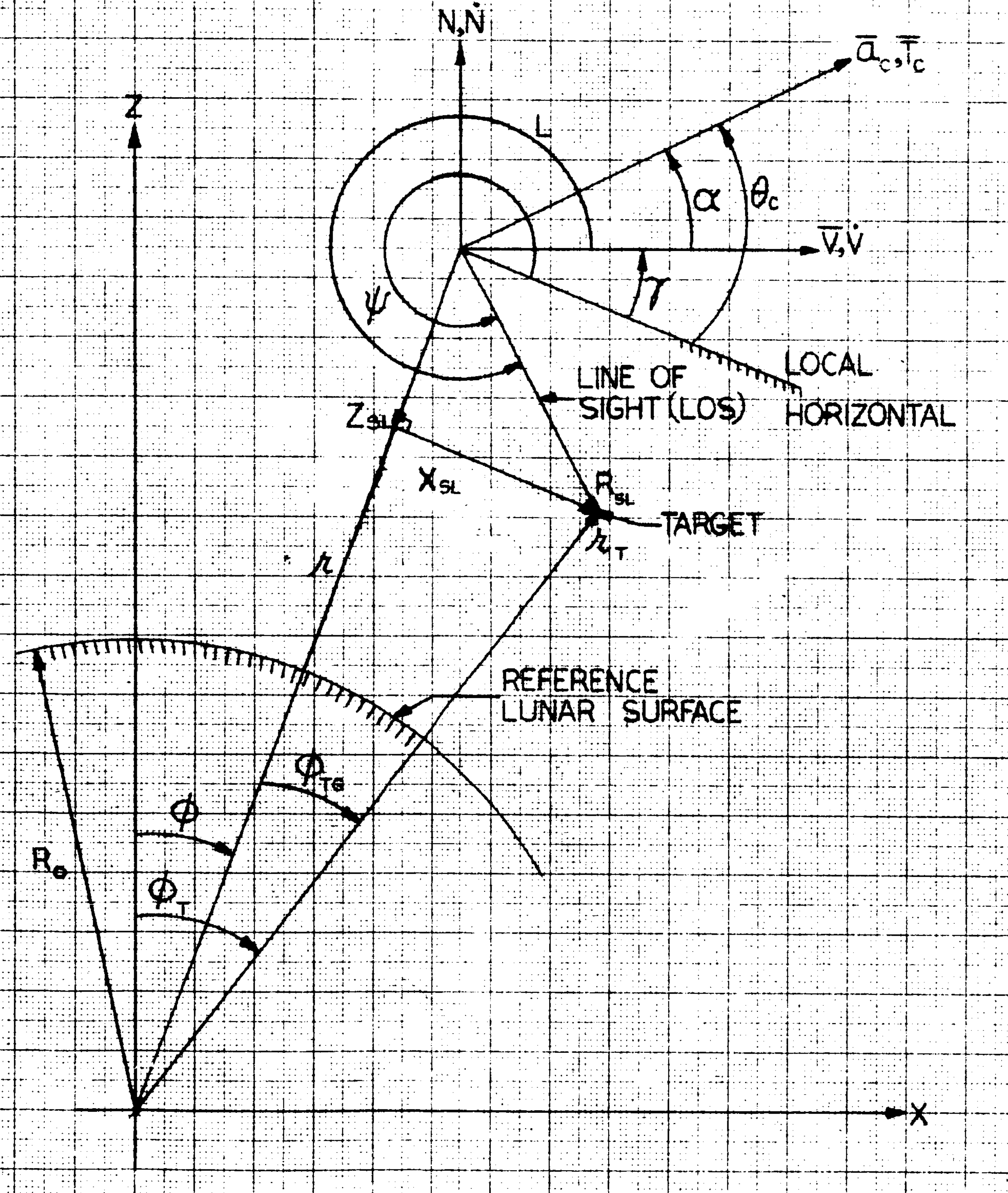
FIG. 6

# GUIDANCE PARAMETERS REQUIRED FOR LOS PROPORTIONAL NAVIGATION

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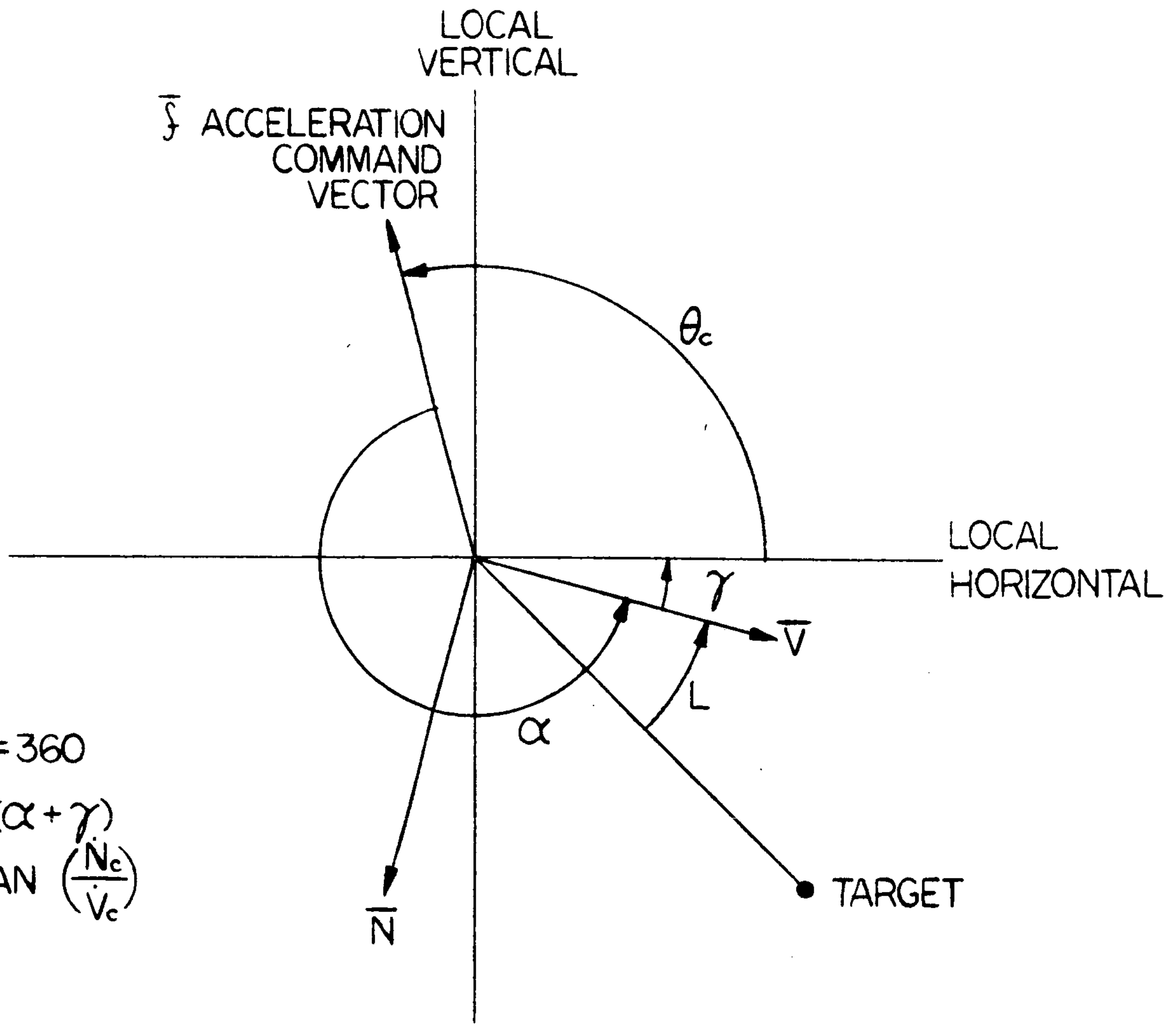


$$\theta_c + \alpha + \gamma = 360$$
$$\theta_c = 360 - (\alpha + \gamma)$$
$$\alpha = \text{ARC TAN} \left(\frac{\dot{N}_c}{\dot{V}_c}\right)$$

FIG 7

DETERMINATION OF $\theta_c$ FROM $\dot{N}_c$ AND $\dot{V}_c$

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FIG. 8

ΔV, INITIAL PITCH ATTITUDE ($\theta_i$), AND FINAL PITCH RATE ($\dot{\theta}_f$) vs INITIAL VELOCITY ($V_o$), AND GUIDANCE GAIN (K), FOR LOS PN

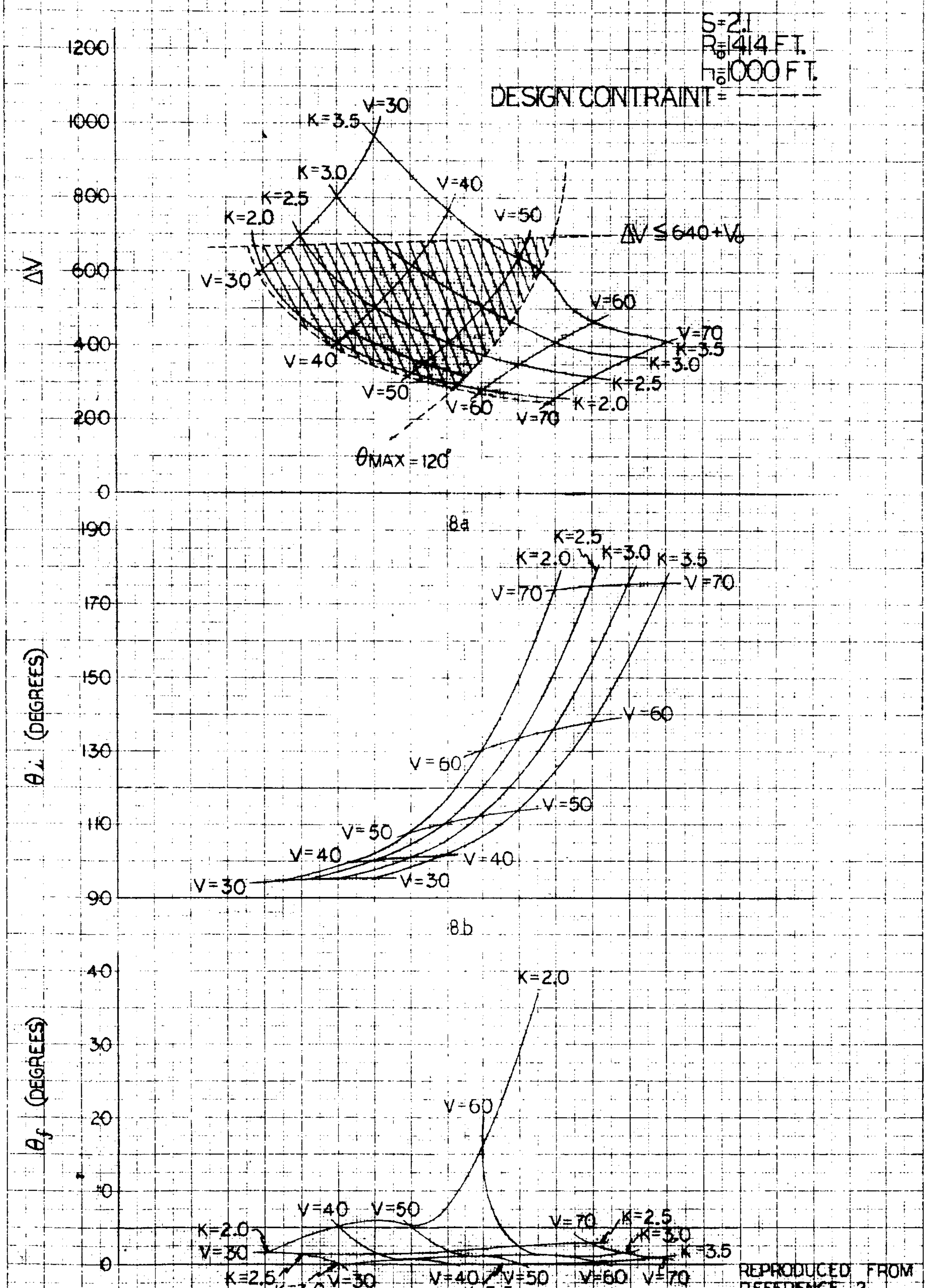

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FIG.9

MLOS PN TRAJECTORY HISTORY

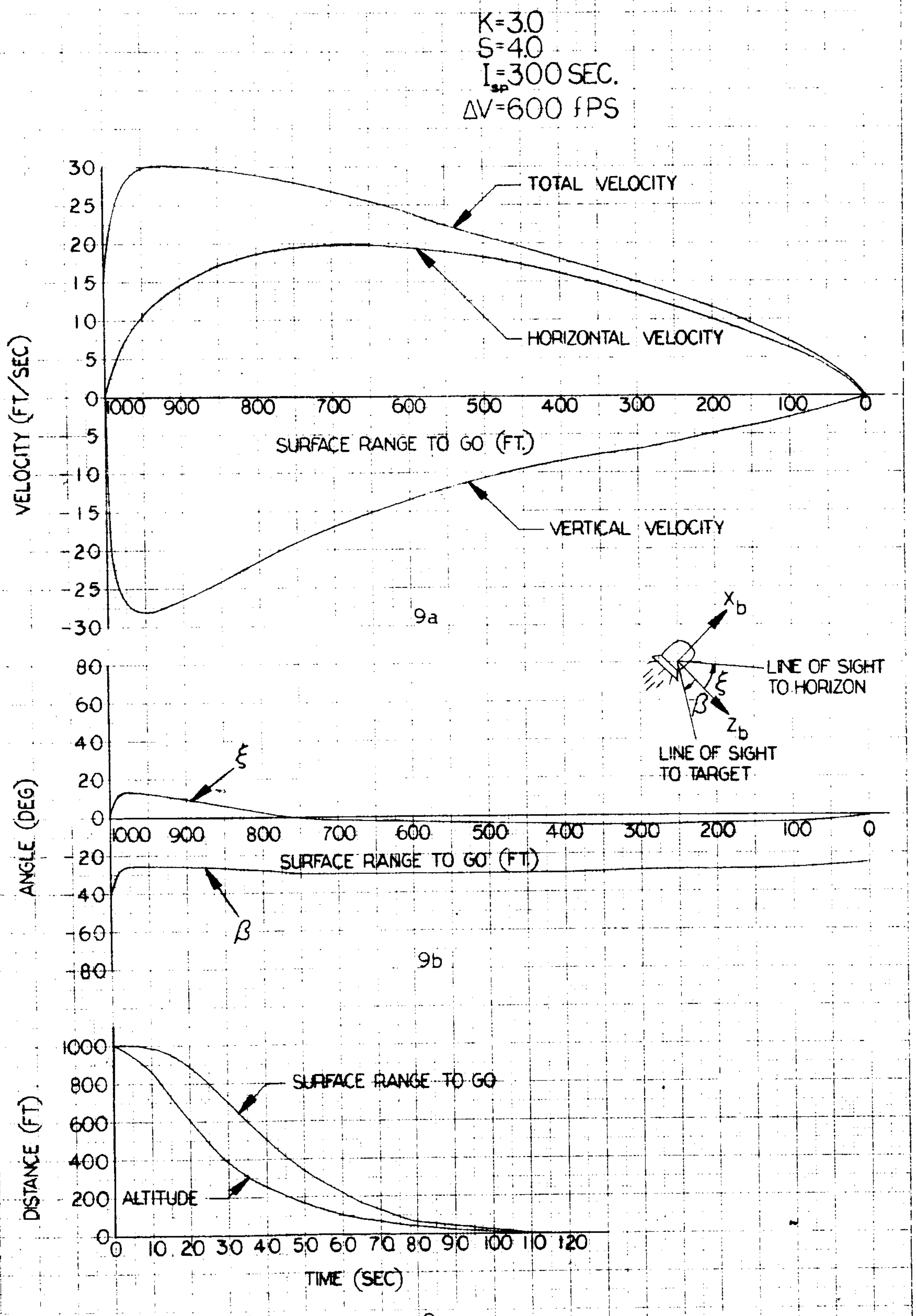


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FIG.9

PITCH, PITCH RATE, AND THRUST PROFILES CORRESPONDING TO MLOS TRAJECTORY OF FIG 9a,b,c

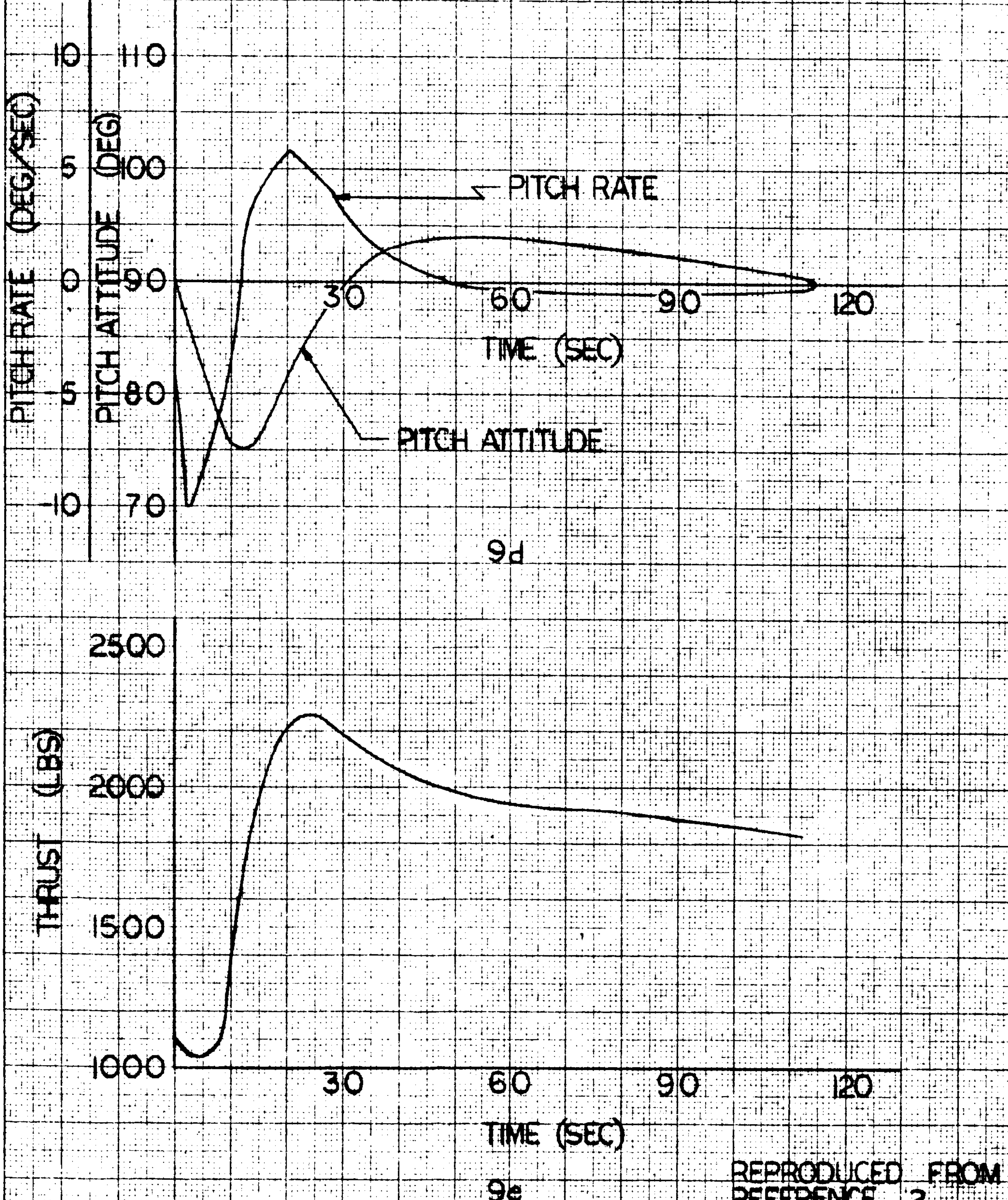


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FIG.10
MLOS PN TRAJECTORY HISTORY

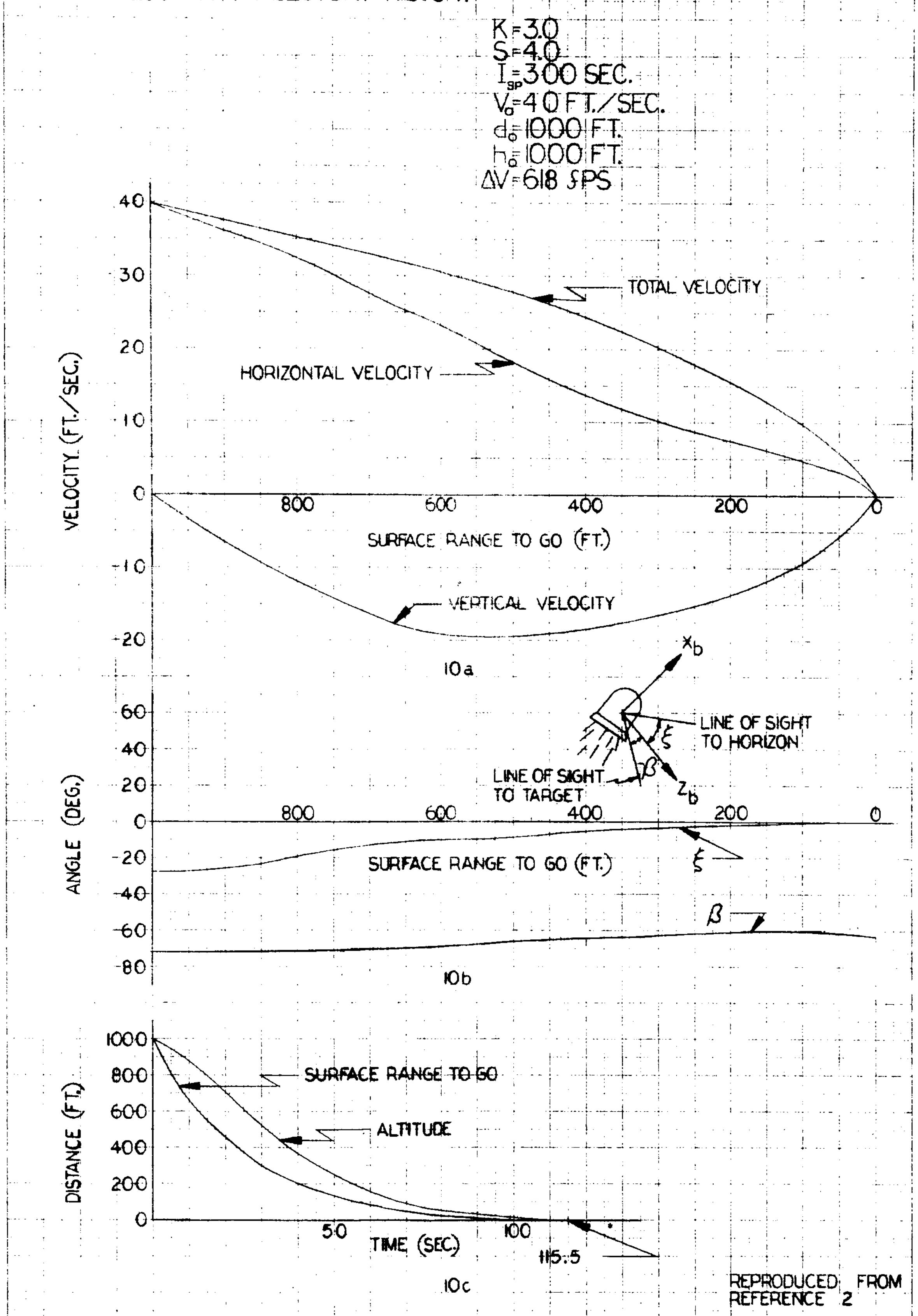


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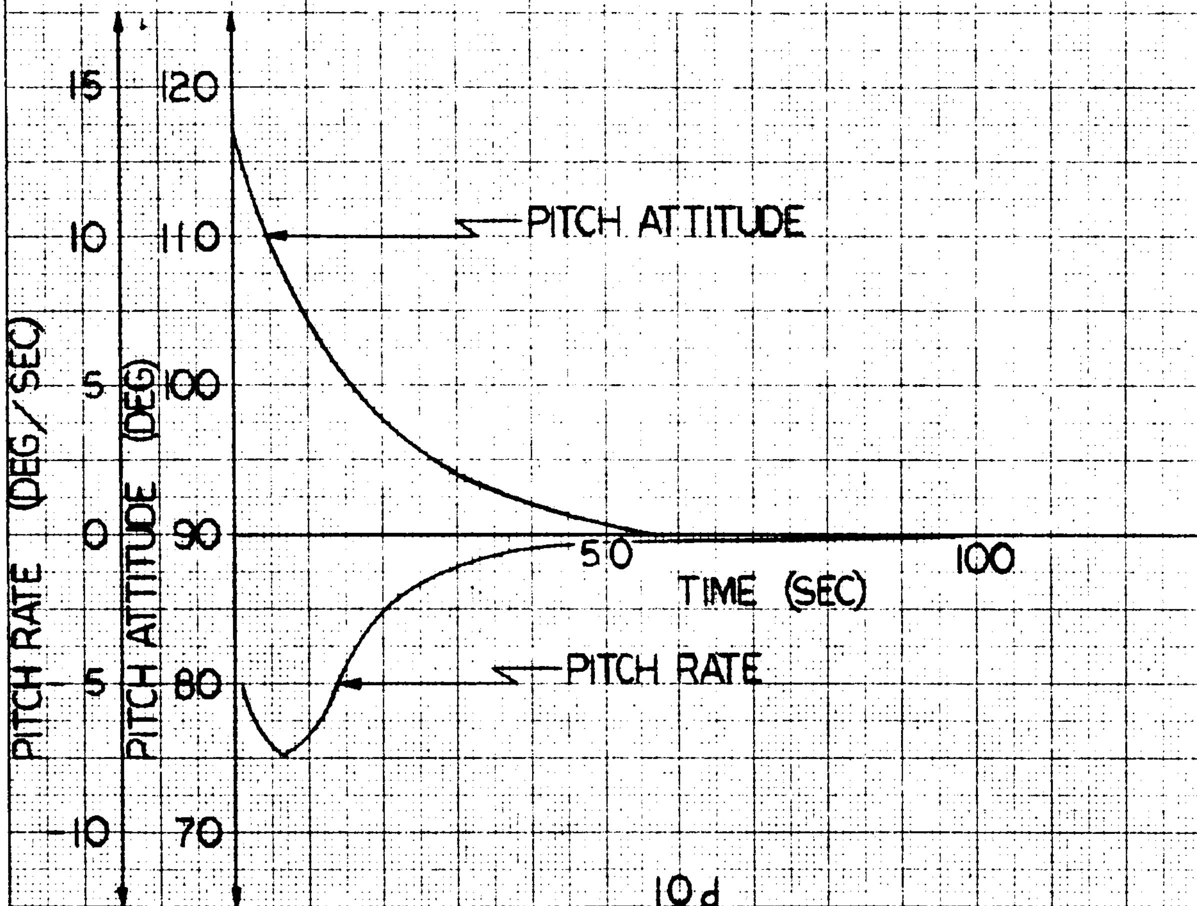
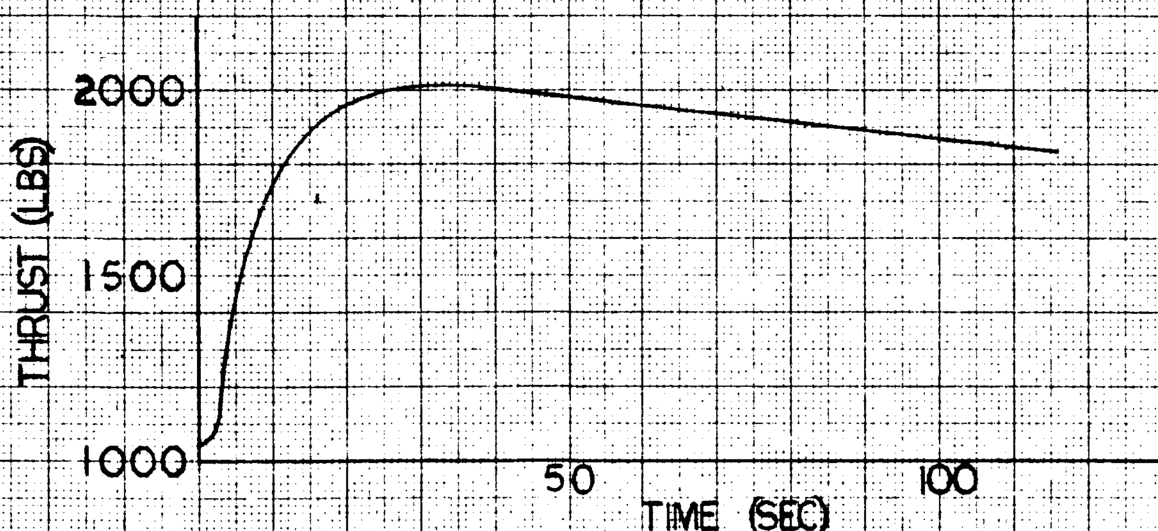
FIG. 10

PITCH, PITCH RATE, AND THRUST PROFILES CORRESPONDING TO MLOS TRAJECTORY OF FIG 10 a,b,c




10d




10e

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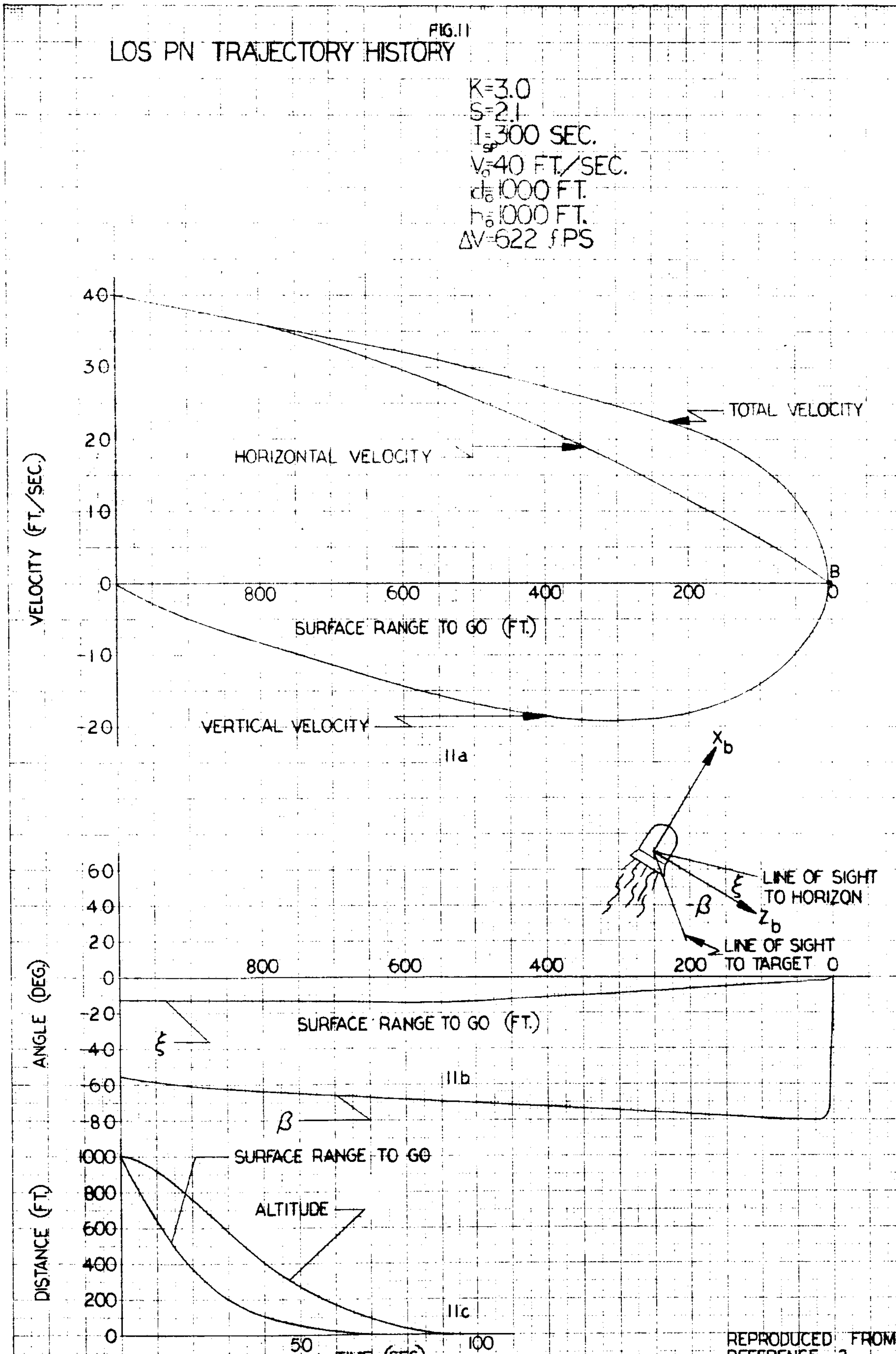


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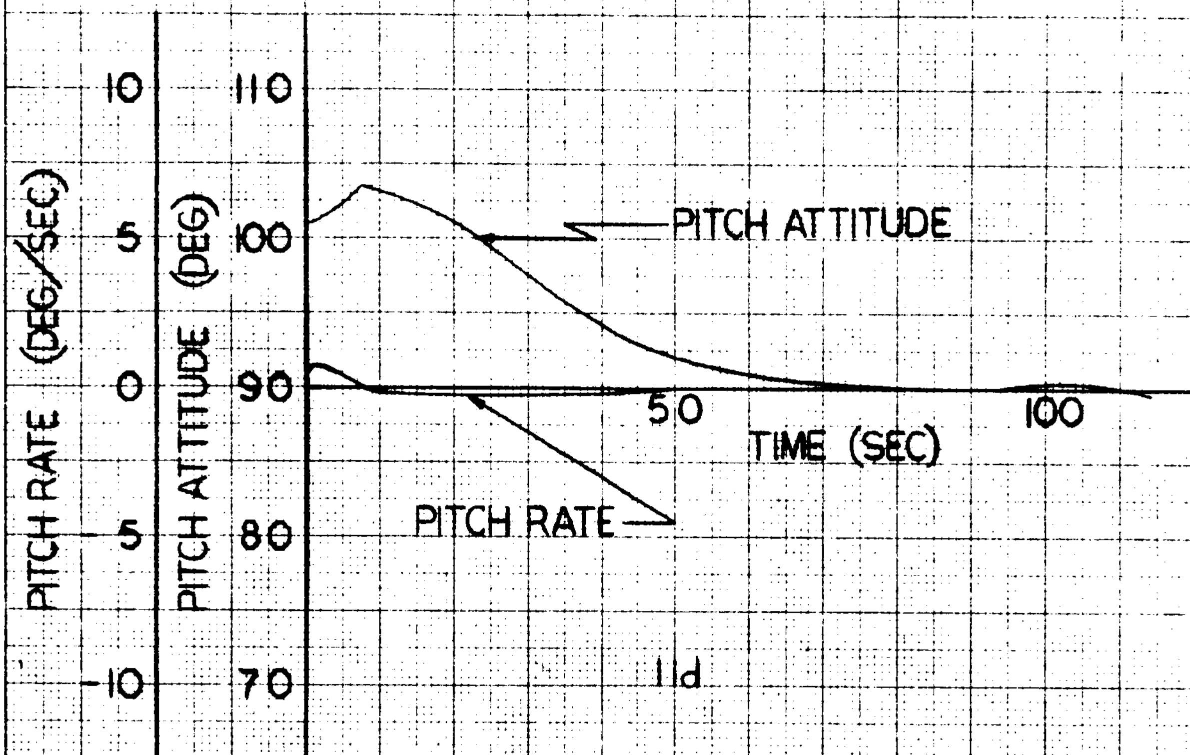
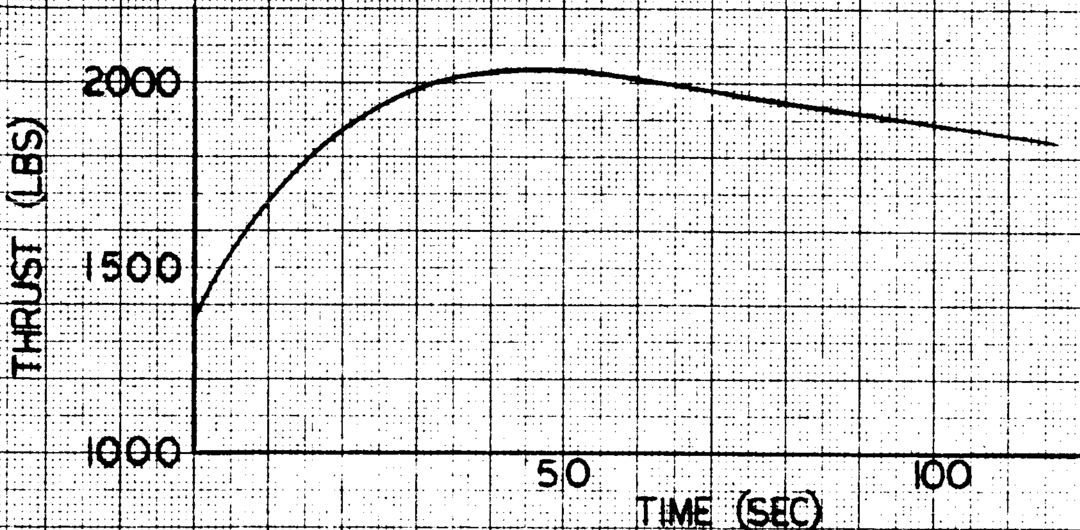
FIG 11

# PITCH, PITCH RATE, AND THRUST PROFILES CORRESPONDING TO LOS TRAJECTORY OF FIG 11a,b,c




11d




11e

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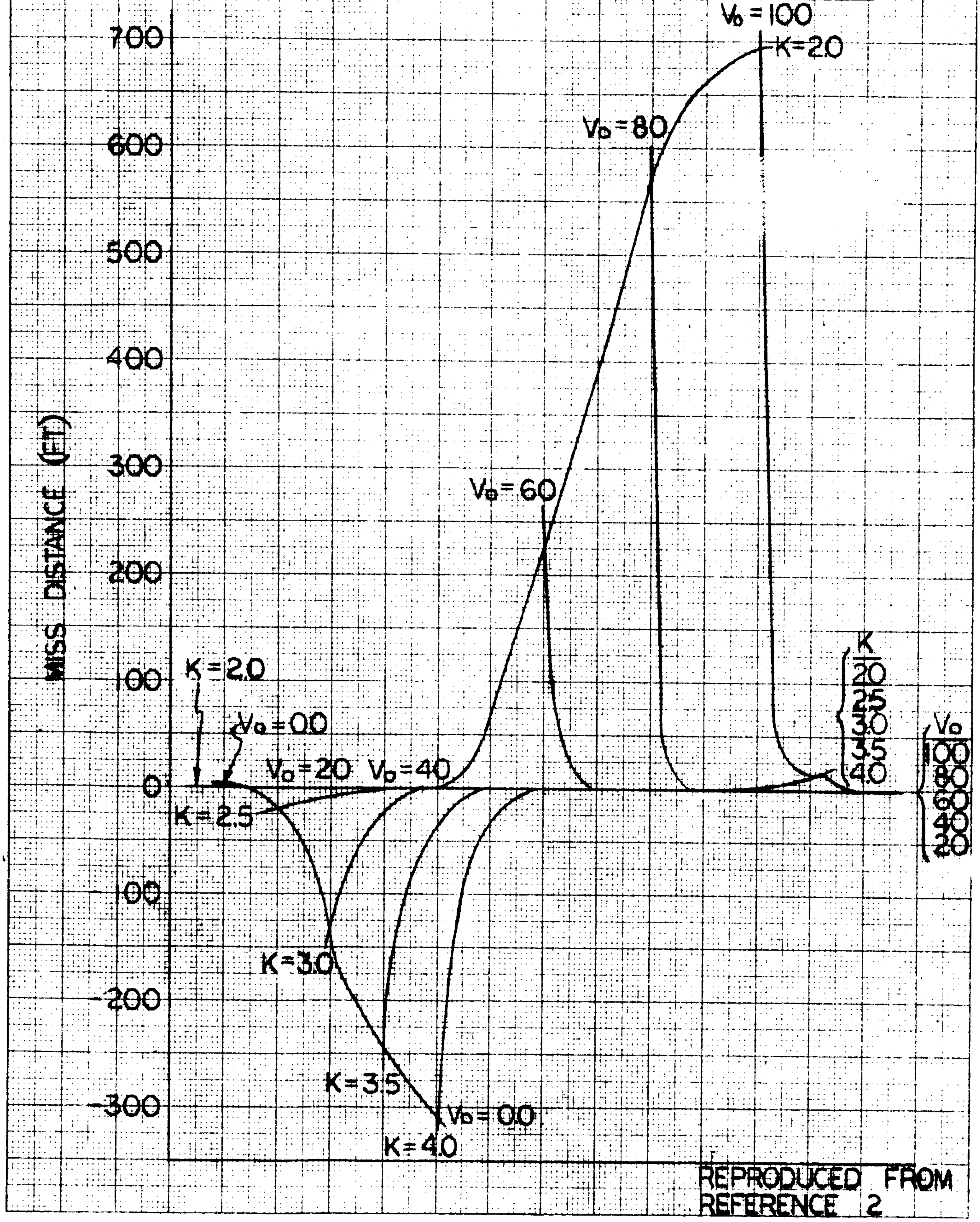


FIG. 12
DOWN RANGE MISS DISTANCES vs K, $V_o$ INITIAL SURFACE RANGE = 1000 FT. S = 21 (MLOS PN)

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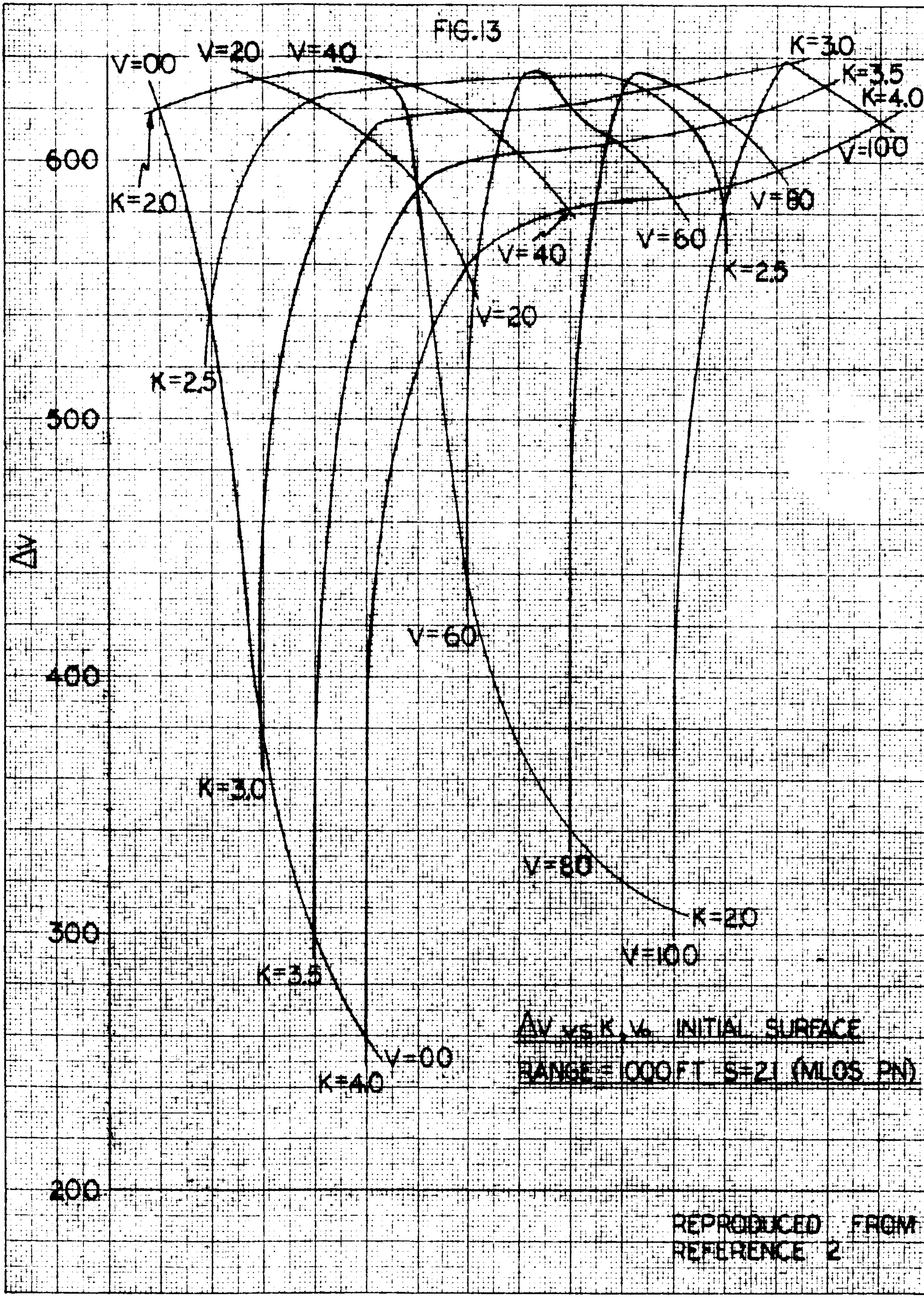
FIG.13
V=00
V=20
V=40
K=3.0
K=3.5
K=4.0
V=100
600
K=2.0
V=80
V=60
V=40
K=2.5
V=20
K=2.5
500
ΔV
V=60
400
K=3.0
V=80
K=2.0
300
V=100
K=3.5
ΔV vs K, V₀ INITIAL SURFACE
V=00
K=4.0
RANGE = 1000 FT S=21 (MLOS PN)
200
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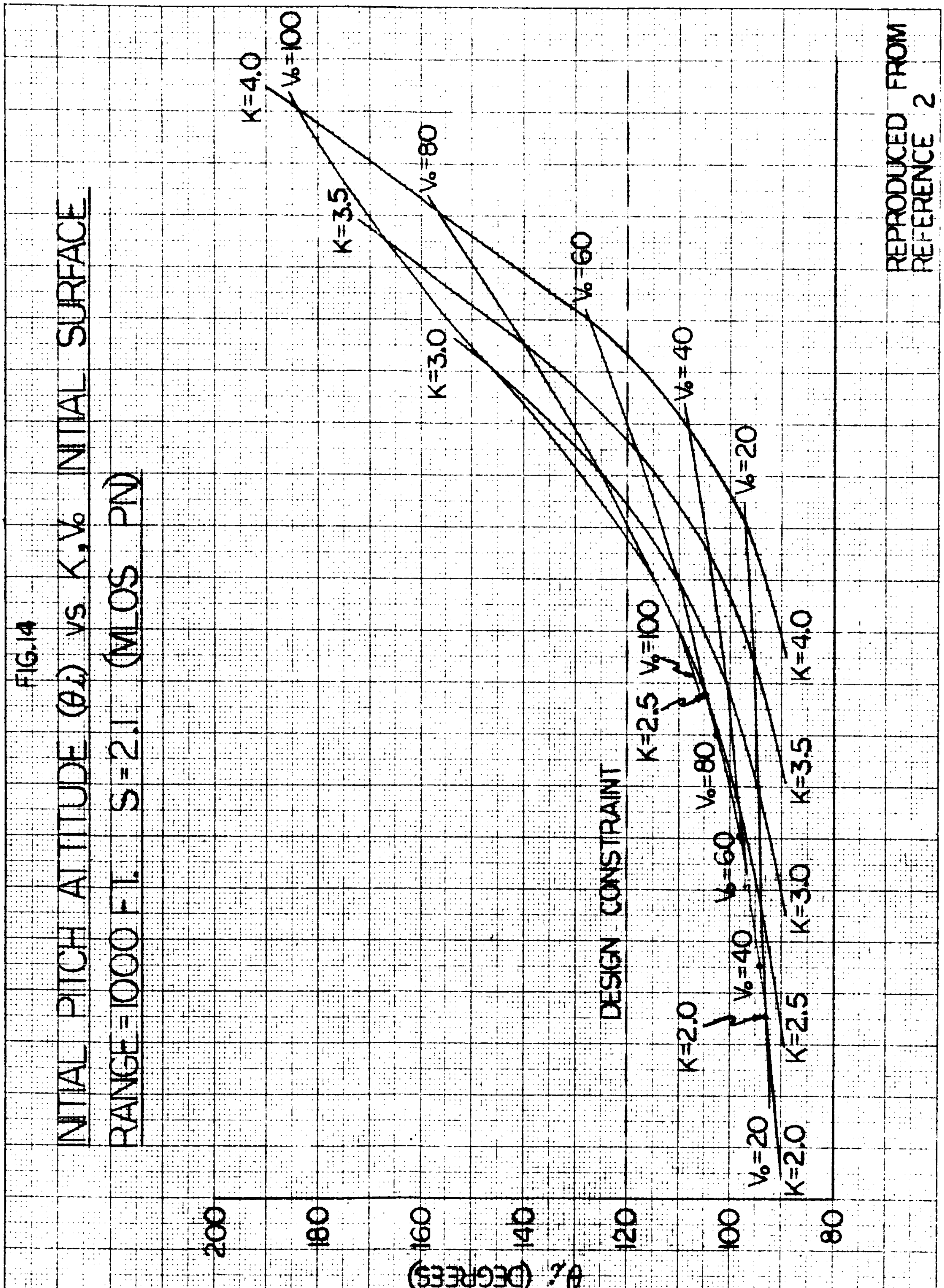

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FIG.15

PITCH ATTITUDE AT TOUCHDOWN ($\theta_f$) vs K, $V_0$

INITIAL SURFACE RANGE = 1000 FT S=2.1 (MLOS PN)

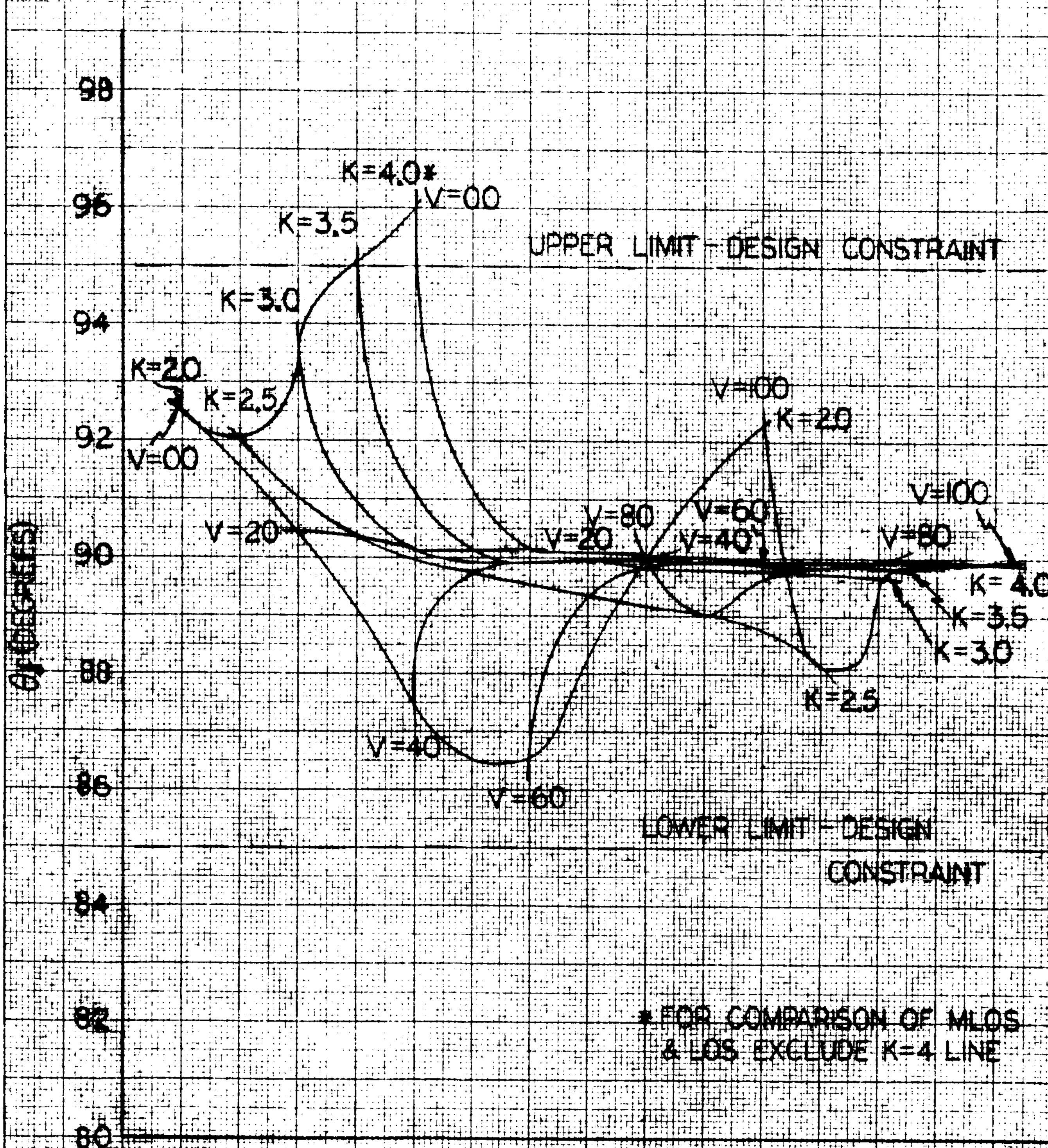


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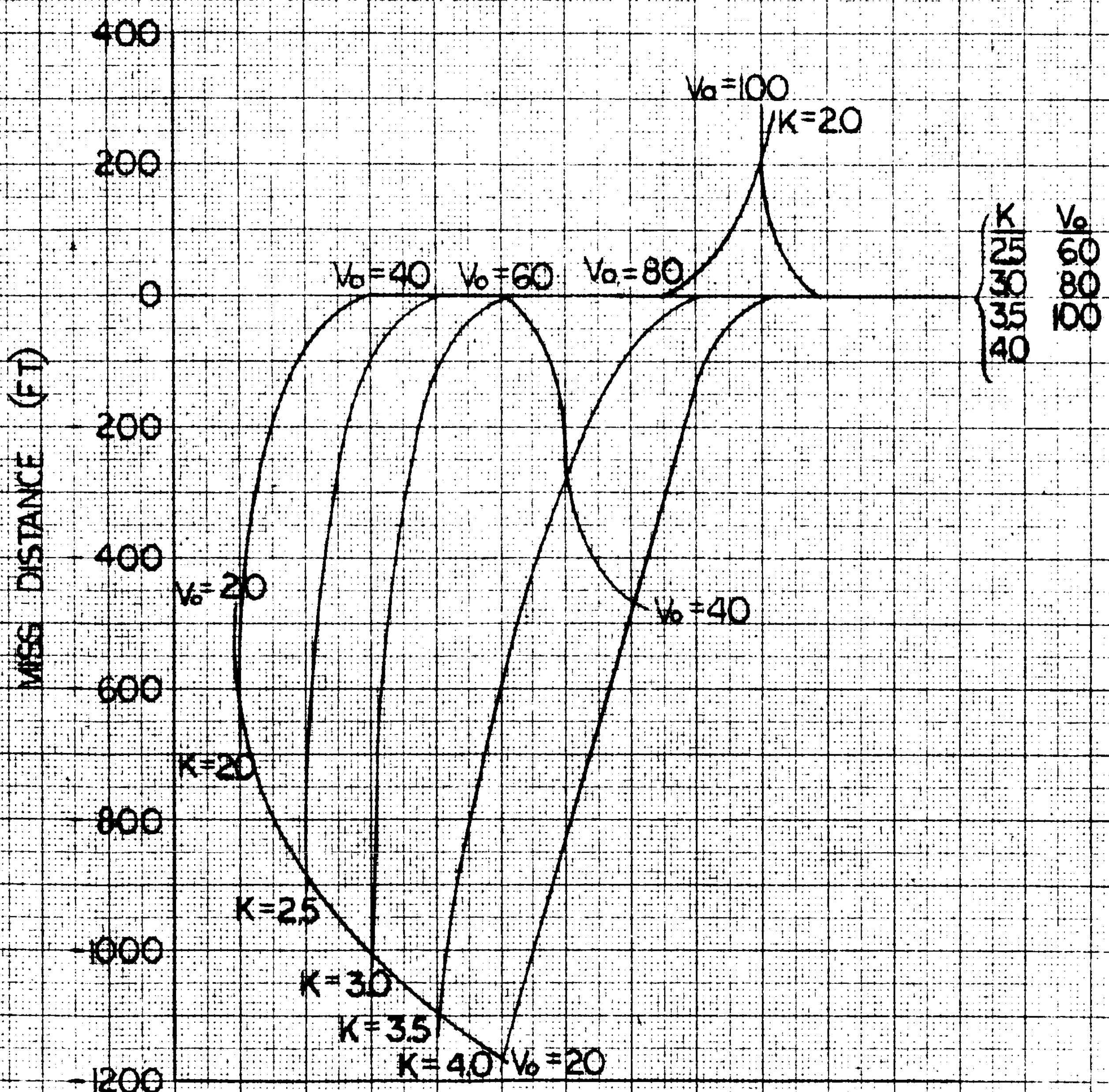
FIG. 16

DOWN RANGE MISS DISTANCES vs K, $V_o$ INITIAL SURFACE RANGE = 2000 FT S = 21 (MLOS PN)

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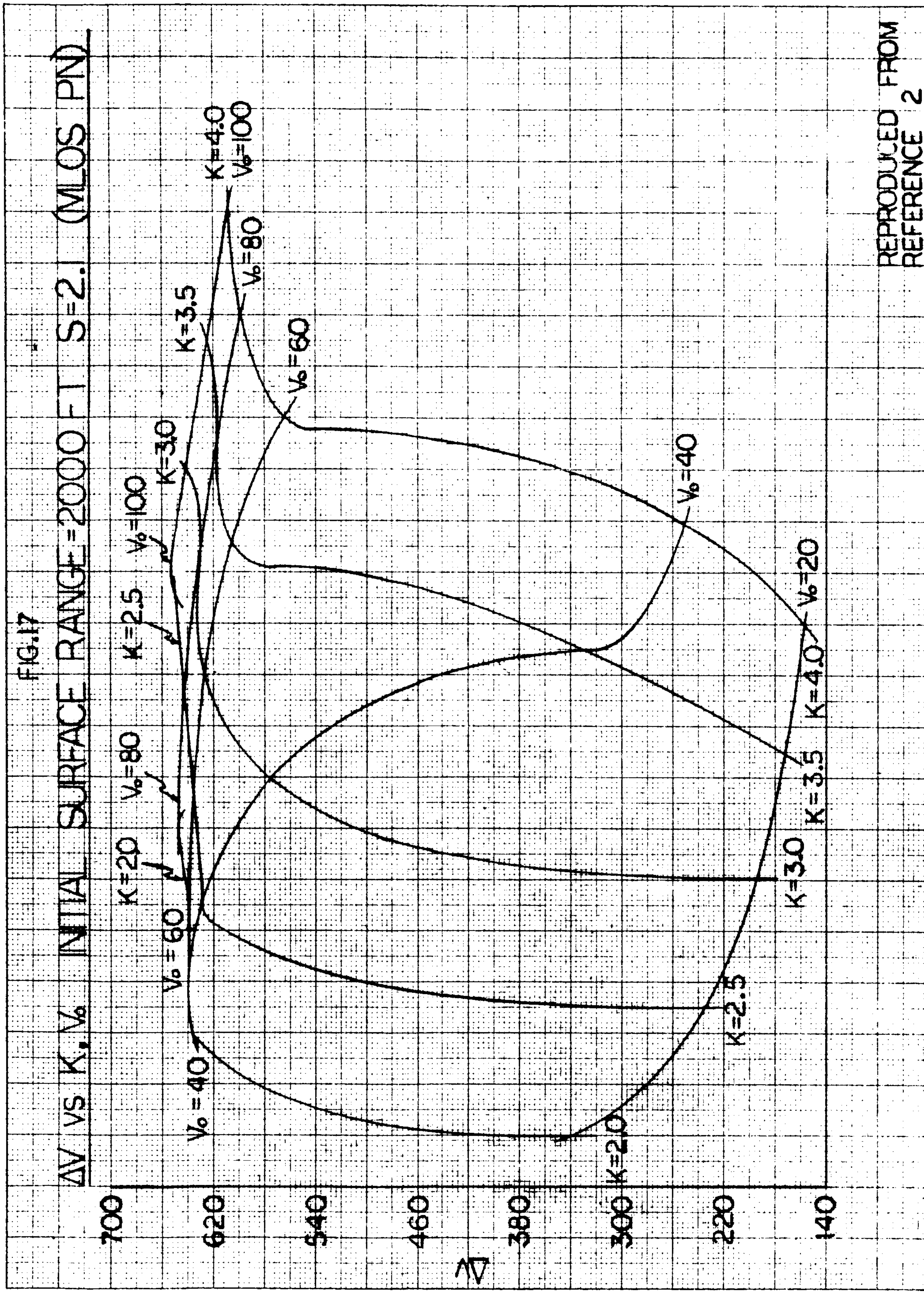

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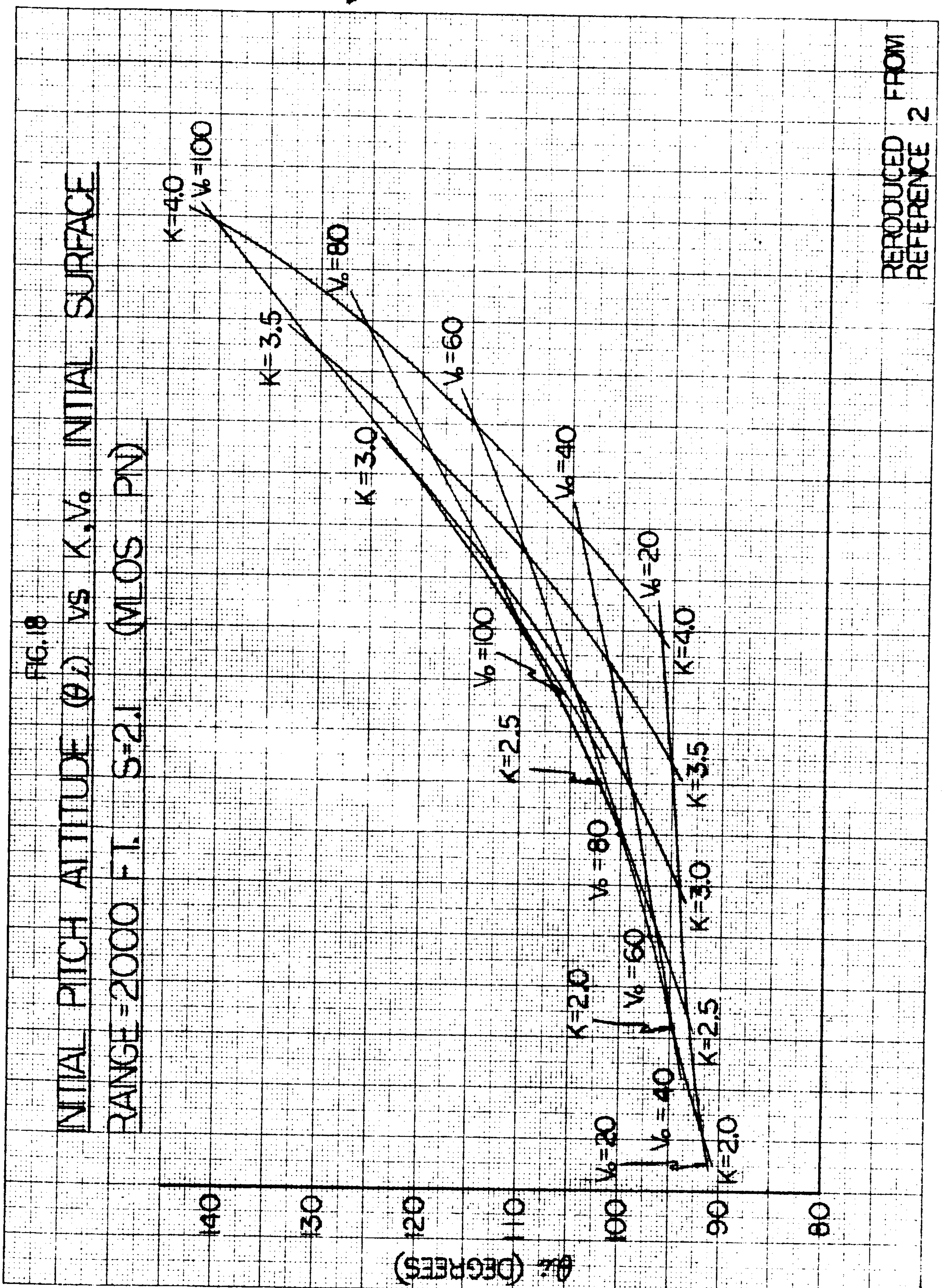

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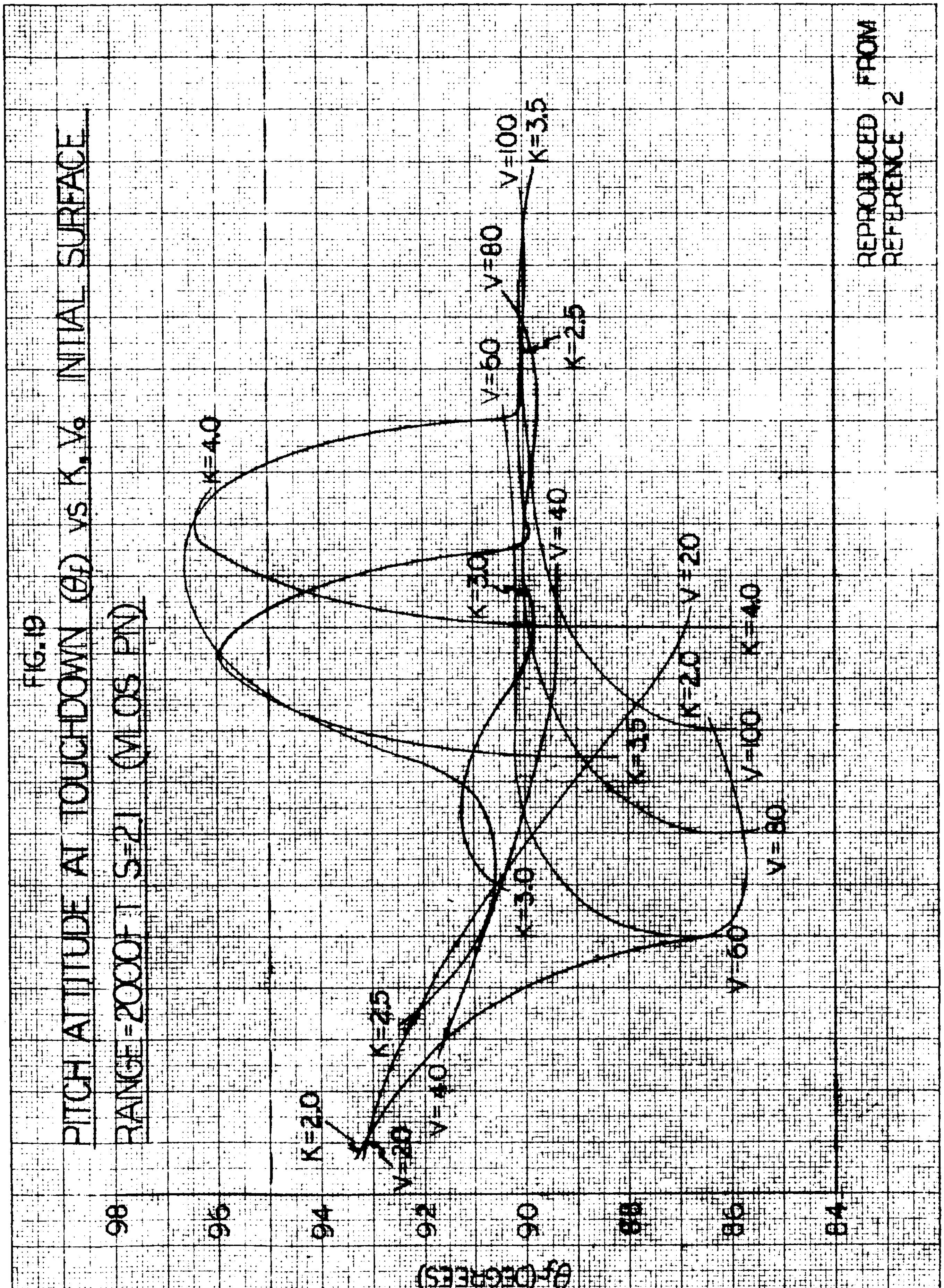


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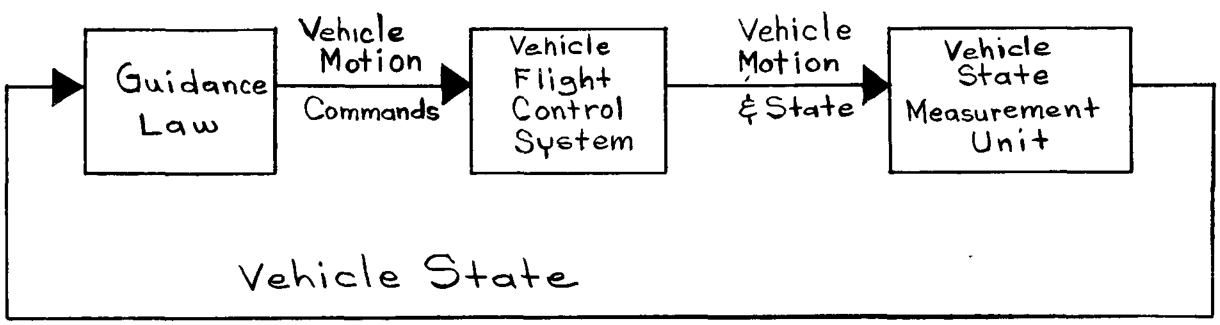
3.7.2 (Continued)

as the extreme cases just covered except with less pronounced affects. As the velocity moves closer toward the lower limit of 0, it will be increasingly more difficult for the Basic Law to perform a landing within the constraints set up, whereas the Modified Law would have less difficulty. As the initial velocity approaches 40 ft/sec, both laws will tend to achieve the target, however the Modified will hit safe touchdown velocities earlier in the trajectory, and exhibit improved visibility characteristics, while approaching at a lower altitude.

## 4.0 POLYNOMIAL GUIDANCE LAW (Ref. 11, 16)

### 4.1 GENERAL DISCUSSION ON IMPLEMENTATION OF POLYNOMIAL GUIDANCE LAW

The simplified diagram for a general closed loop, self contained guidance law implementation is shown in Figure 20.




Simplified Diagram of Self Contained Guidance Law (Fig. 20)

The fundamental elements are:

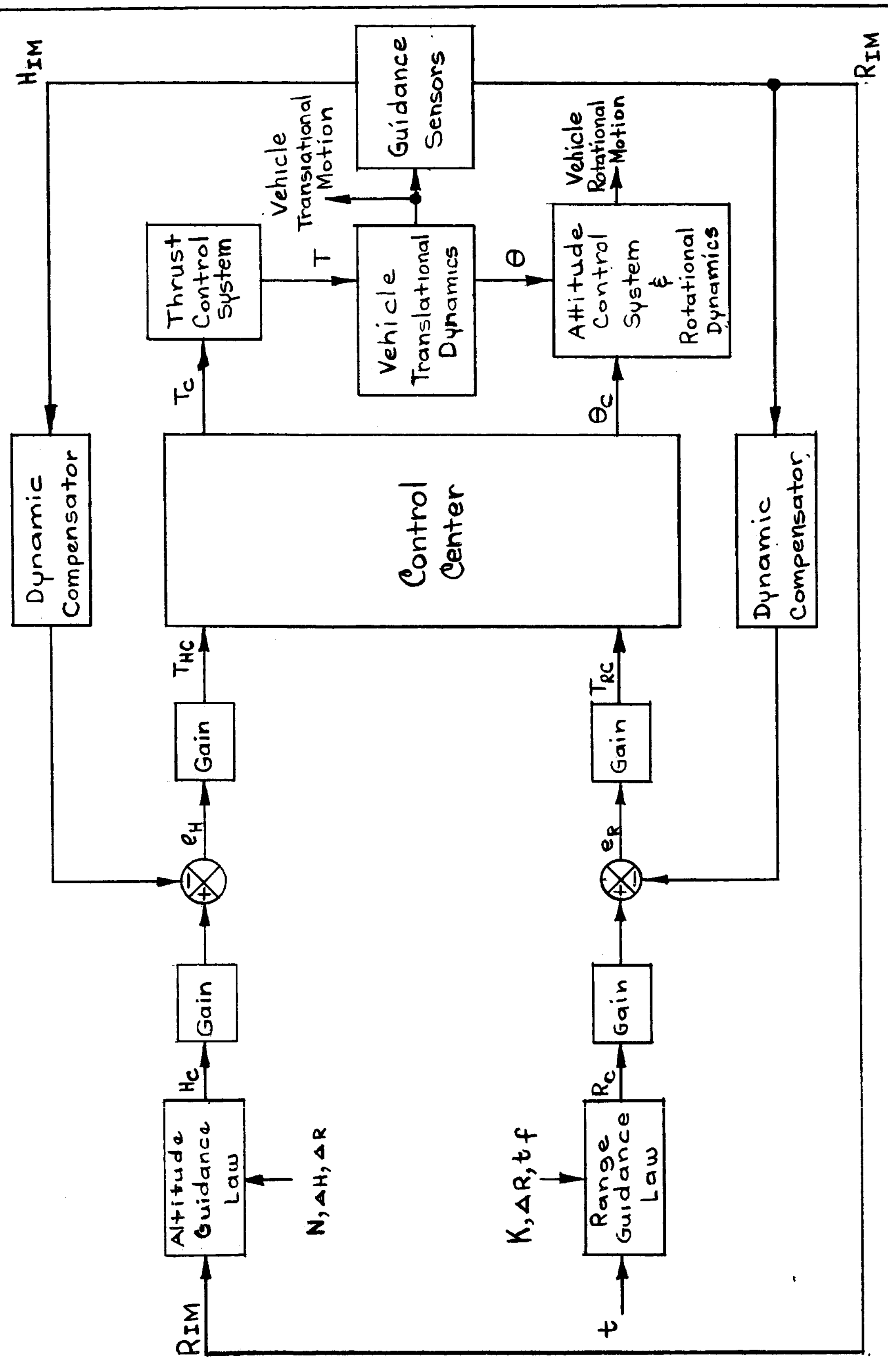
(1) The guidance law, i.e., the criteria upon which commands to the vehicle based on vehicle existing and desired state are generated.

(2) The vehicle response (translation and rotation) to commands and its resulting state.

(3) The measurement of vehicle state by some type of measurement unit.

(4) Closing the loop by reporting back to the guidance law, the vehicle state so that the guidance law may generate updated commands.

The Polynomial Guidance Law implementation falls under the same general pattern as discussed above, but is shown in more detail in Figure 21.




POLYNOMINAL LAW LUNAR LANDING
BLOCK DIAGRAM
FIG. 21

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4.1 (Continued)

The basic Polynomial Guidance Law consists of two parts, both fashioned in the general pattern indicated in Figure 20 and described by equations (24) and (27) and (20) in the next section 4.2. The two parts consist of a range guidance law and an altitude guidance law.

The range guidance law (equation 24) varies as a function of time (t) and depends on the prespecified time of flight ($t_f$) and the downrange distance to desired aimpoint ($\Delta R$). K is a guidance gain constant which specifies the Range-time trajectory characteristic. The range guidance law output is the range command ($R_c$).

The altitude guidance law (equation (20) in simplified form and equation (27) in final form) is dependent on the instantaneous measured downrange distance ($R_{IM}$) and in addition depends on the initial altitude above the target point [$\Delta H$ or $h- (h_{LG} + H_{SW})$] (see table in Section 4.2.8 for symbol definitions). N is a guidance gain constant which controls the trajectory shape ($h_c$ vs. $R_{IM}$). The output of the altitude guidance law is the altitude command $h_c$.

The following discussion refers to Figure 21. In sequence of occurrence, the range command is varied as time increases. This causes vehicle range to vary and then the altitude commanded varies, so that the range portion of the guidance law takes the initiative and the altitude command follows changes in range. Both the range and altitude commands (Rc, Hc) after going through a gain adjustment are compared with their respective measured quantities; i.e. vehicle range ($R_{IM}$) and vehicle measured altitude ($H_{IM}$). As a result range error ($e_R$) and altitude error ($e_H$) signals are determined. The measured range and altitude had been previously fed through a dynamic compensator (stability compensation network to aid guidance loop stability) before comparison. The error signals in altitude and range are adjusted in gain and the output range and altitude thrust command components ($T_{Hc}$, $T_{Rc}$) are proportional to the error components.

For an ideal system with no limits on attitude or thrust magnitude, all that would be required would be to convert the rectangular thrust vector components to a polar coordinate thrust vector having a magnitude and direction. This would result in determining thrust magnitude commands ($T_c$) and vehicle attitude commands ($\theta_c$) to orient the thrust vector. The equations below accomplish the conversion.

4.1 (Continued)

$$T_c = (T_{H_c}{}^2 + T_{R_c}{}^2)^{\frac{1}{2}} \qquad (14)$$

$$\theta_c = \tan^{-1} \left[ T_{H_c} / T_{R_c} \right]$$

The action described by equation (14) would occur in the control center of Figure 21, if there was no restrictions in attitude and thrust. However, these restrictions will complicate the control center still further. (See ref. 16 for further discussion.)

The guidance law control loops constructed by the state and command comparison produces a thrust and attitude command proportional to error. The attitude loop formed may be recognized as basically a type 2 control system (due to vehicle inertia) where a steady accelleration output requires a constant actuating error signal and error is fully nulled with a constant velocity input. Both range and altitude loops are complicated by the fact that the rectangular to polar transformation must be performed and therefore there is an interaction between the loops.

Analysis(using the root locus) and simulation has shown that each loop is stable when decoupled. However, when the loops are coupled in an actual simulation, instability occurs due to the coupling of the slow acting attitude loop (for large commands) with the range loop. Stability was obtained by an empirical method which limited the vector difference between that which is commanded and that which is provided by the vehicle.

Referring again to Figure 21, the attitude command is fed to a simulated attitude control system which includes vehicle rotational dynamics. The output of the vehicle is rotational motion. Both vehicle thrust and attitude outputs are fed into the simulated vehicle translational dynamics. The output of this is vehicle translational motion. The translational motion is sensed by the vehicle navigational sensors and the resulting outputs are measured range ($R_{IM}$) and measured altitude ($H_{IM}$). These are fed through the dynamic Compensators which close the altitude and range loops of the guidance law as previously described.

4.2 THEORY AND DEVELOPMENT OF POLYNOMIAL GUIDANCE LAW
(A Guidance Law for Automatic Lunar Landing)

4.2.1 Introduction (Ref. 11, 16)

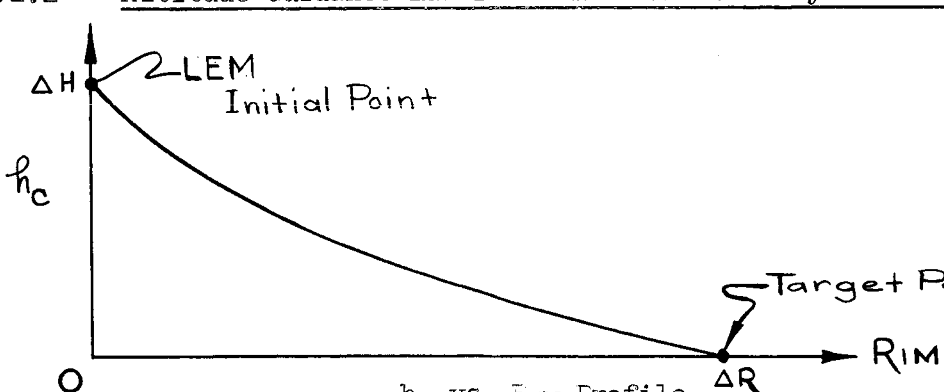
The Polynomial Guidance Law was developed at GAEC by H. Sperling & M. Rimer for the automatic hover to touchdown maneuver. Selection of the composition of the guidance law was based on the supposition that range commands should be determined from initial target downrange distance and some quantity which reflects time used from a preselected time of flight. This would constrain range commands to start at vehicle initial range and gradually approach the desired range as preselected time of flight is approached.

Another supposition was the altitude commands should be controlled as a function of target initial altitude and range left to go. This constrains the altitude command to approach the desired value as the desired range is approached.

The above assumptions, when properly implemented, were the basic factors in the development of a satisfactory guidance law. The proper implementation insured that the range command be proportional to target range and that the vehicle range command increases (range being measured from the initial starting point) as time increases toward the final determined time of flight. Proper implementation also insures that the altitude command would decrease to the desired value as the downrange target is approached.

The complete Polynomial Guidance Law consists of two command equations; one for altitude command and one for range commands. The altitude guidance law is determined first, the range next. See list of symbols in section 4.2.8 for definitions.

4.2.2 Altitude Guidance Law Determination for Polynomial Guidance




$h_c$ vs. $R_{IM}$ Profile
Figure 22

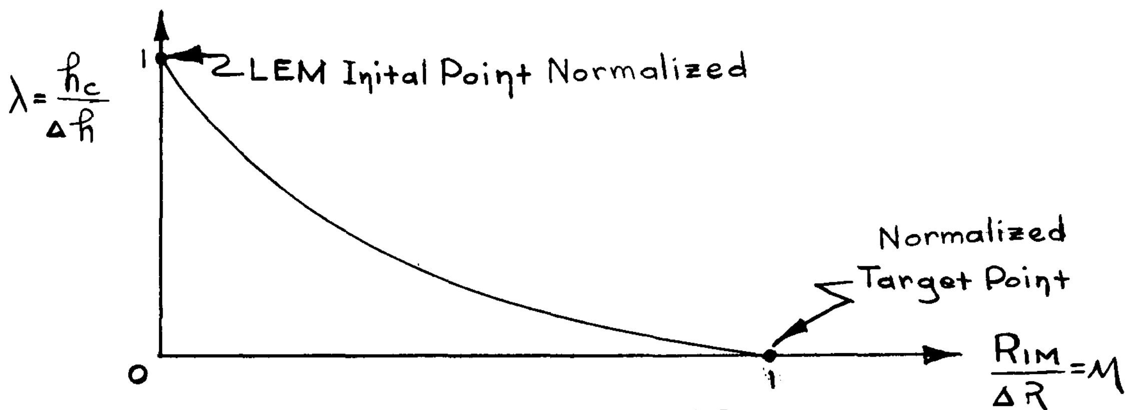
4.2.2 (Continued)

A typical desirable commanded altitude ($h_c$) vs. actual range ($R_{IM}$) trajectory is shown in Figure 22 (see further discussion). The boundary conditions which must be satisified are:

at: $R_{IM} = 0$ , $h = \Delta H$

$R_{IM} = \Delta R$ , $h = 0$

The curve may be normalized with respect to target altitude and range so that the ordinate is represented by $\frac{h_c}{\Delta h}$ and the abscissa by $\frac{R}{\Delta R}$ as illustrated in figure 23.




Normalized $h_c$ vs. $R_{IM}$ Profile

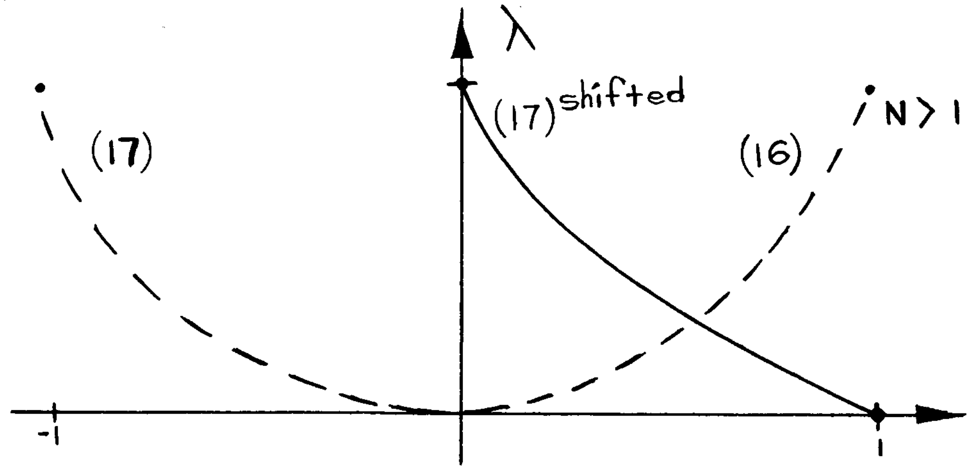
Figure 23

Letting $\mu = \frac{R_{IM}}{\Delta R}$ and $\lambda = \frac{h_c}{\Delta h}$, boundary conditions for figure 24 are as follows:

$$\text{at } \mu = 0, \ \lambda = 1 \tag{15}$$

$$\text{at } \mu = 1, \ \lambda = 0$$

4.2.2 (Continued)




Attitude Guidance Eq. Determination
Figure 24

It is necessary to obtain a function which describes the variation between $\lambda$ and $\mu$ as illustrated in Figure 23 satisfying the boundary conditions. Define a function as follows: (See Fig. 24)

$$\lambda(\mu) = \mu^N \quad \text{for } 0 \leqq \mu \leqq 1 \qquad (16)$$

$$\lambda(\mu) = (-\mu)^N \quad \text{for } 0 \leqq \mu \leqq -1 \qquad (17)$$

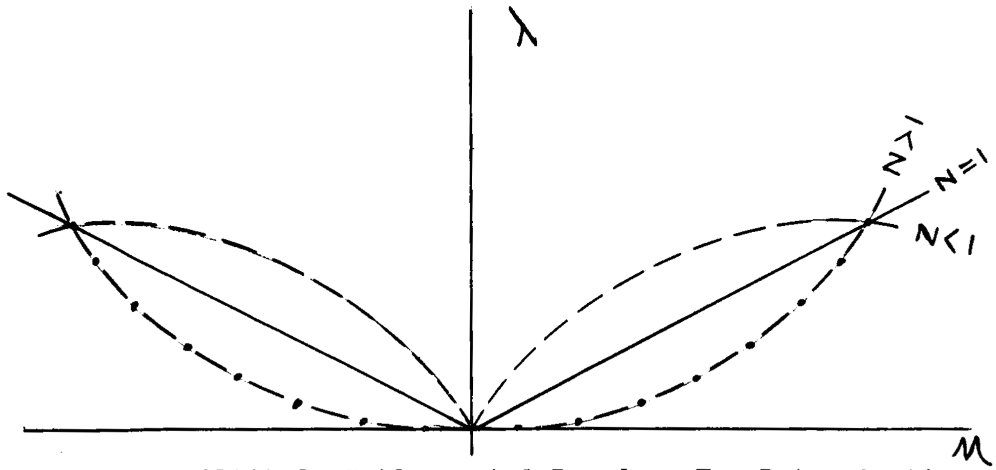
This defines an even function regardless if N is odd or even.

In order to obtain the desired function, part (17) of the above defined function, is shifted to the right by one unit. This is performed by making the substitution $\mu = \mu - 1$ in (17) as follows:

$$\lambda(\mu) = (-(\mu - 1))^N = (1 - \mu)^N \qquad (18)$$

The boundary conditions (15) are satisfied by equation (18).

4.2.2 (Continued)




Altitude Guidance And Boundary Eq. Determination
(Figure 25)

To obtain a trajectory of a desired general shape requires proper selection of the value of N. These are indicated in Fig. 25 (the shape of the function is observed in the ranges from N = 0 to N > 1). They are as follows:

| | |
|---|---|
| for $N > 1$ | a concave (or sagging) trajectory is obtained |
| for $N = 1$ | a linear (straight line) trajectory is obtained |
| for $N < 1$ | a convex trajectory is obtained |

The guidance law equation, upon substitution for the variables $\lambda$ and $\mu$ in (18) becomes $hc = \Delta H \left(1 - \frac{RIM}{R}\right)^N$ (19)

4.2.3 Generalized Altitude Portion of Polynomial Law

Although the guidance law derived above and studied in detail in Ref. 16 does not have the complete capability of producing a trajectory of any possible shape, it is possible to extend the guidance law to a generalized form which approaches this capability. The generalized function may be obtained by applying a similar procedure to the one used previously.

The desired expression is obtained by adding all terms of lower order to the $N^{th}$ term (see (18)) forming the general polynomial as follows:

$$\lambda(\mu) = d_p(-\mu)^N + d_{p-1}(-\mu)^{N-1} + \dots + d_1(-\mu) + d_o(-\mu)^o$$

Shifting one unit to the right $\mu = \mu - 1$

$$\lambda(\mu) = d_p(1-\mu)^N + d_{p-1}(1-\mu)^{N-1} + \dots + d_1(1-\mu) + d_o$$

$$\lambda(\mu) = \sum_{i=0}^{P} d_i \left(1 - \frac{RIM}{R}\right)^{Ni}$$

Substitution for the variables results in

4.2.3 (Continued)

$$\frac{h_c}{\Delta H} = \sum_{i=0}^{P} d_i\left(1 - \frac{R_{IM}}{\Delta R}\right)^{N_i}$$

or

$$h_c = \Delta H \sum_{i=0}^{P} d_i\left(1 - \frac{R_{IM}}{\Delta R}\right)^{N_i} \qquad (20)$$

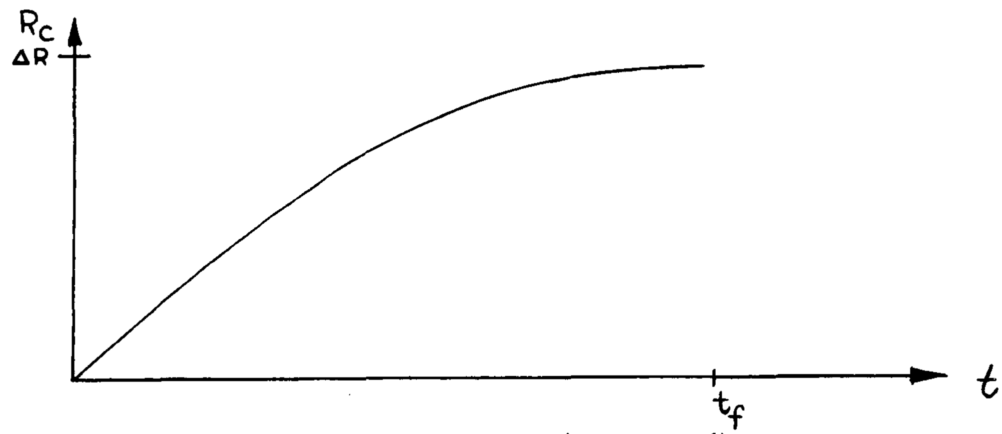
$\sum_{i=1}^{P} d_i = 1$ since the boundary conditions $\frac{R_{IM}}{\Delta R} = 0$, $\frac{h_c}{\Delta H} = 1$

must be satisfied

Equation (20) is the expression for the generalized guidance law, which however was not carried any further since it appeared that guidance law (18) with $N > 1$ was sufficient for the study.

4.2.4 Range Guidance Law Determination

A typical desireable range vs. time trajectory is shown in Fig. 26.




Range Vs. Time Trajectory (Figure 26)

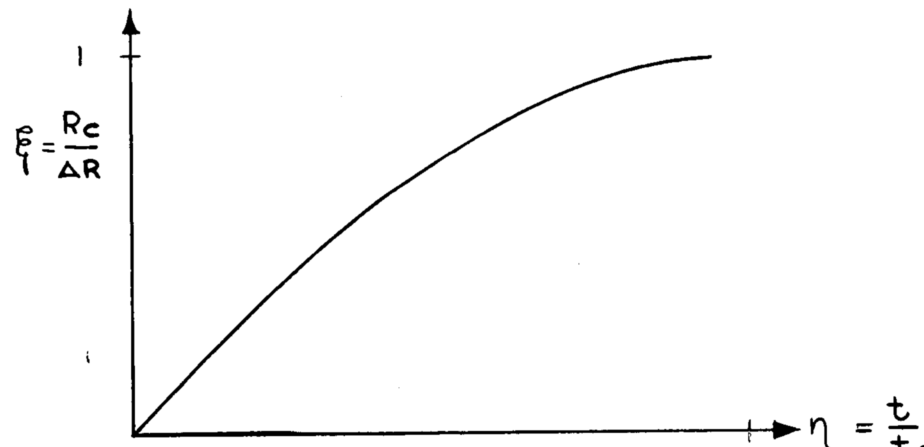
It satisfies the required boundary conditions that at $t = 0$, $R_c = 0$ and at $t = t_f$, $R_c = \Delta R$. In addition the curve exhibits a monitonically decreasing range velocity command (slope decreasing) as time increases and the final range velocity commanded $\left(\frac{dR_c}{dt}\right)$ approaches zero as t approaches $t_f$. Note that a downrange velocity (other than zero) exists at $t = 0$, since the slope of the function is not zero.

In a manner similar to the procedure used for determining the

4.2.4 (Continued)

altitude function, the range function may also be determined as follows:

Step (1) Normalize the curve in Fig. 26, which becomes Fig. 27. Let $\xi = \frac{R_c}{\Delta R}$ and $\eta = \frac{t}{t_f}$




Normalized Range Function
(Figure 27)

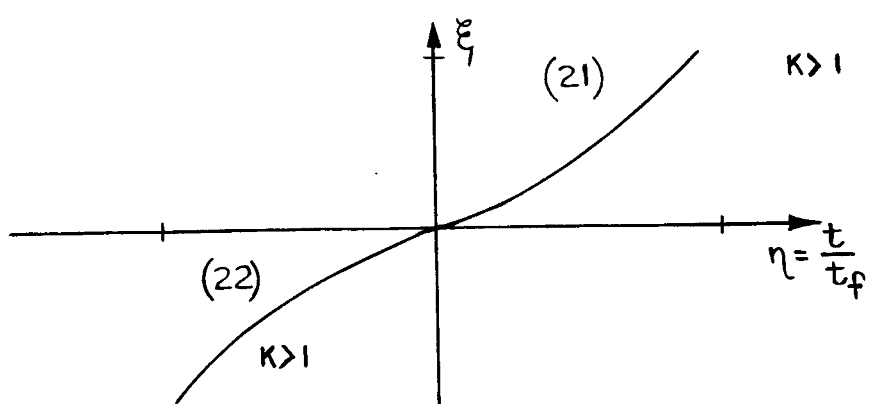
Step (2) Define the following function

$$\xi = \eta^K \qquad 0 \leqslant \eta \leqslant 1 \qquad (21)$$

$$\xi = -(-\eta)^K \qquad -1 \leqslant \eta \leqslant 0 \qquad (22)$$

$$K > 1^*$$

Equation (22) has the desired shape characteristic (See Fig. 28), but must be repositioned to satisfy the required boundary conditions




Range Guidance Eq. Determination (Fig. 28)

* $K > 1$, since $K \leqslant 1$ would produce a slope which does not approach zero, as $\eta$ increases in (22).

4.2.4 (Continued)

Step (3) Repositioning of the function is performed by shifting the function (22) to the right by one unit (perform the substitution $\eta = \eta - 1$) and then shifting the function in a positive direction by one unit (perform the substitution $\xi = \xi - 1$). Perform as follows:

$$\xi - 1 = -\left[-(\eta - 1)\right]^K$$

$$\xi = 1 - (1 - \eta)^K \qquad 0 \leq \xi \leq 1 \qquad (23)$$

The required boundary conditions are

$$\text{at } \eta = \frac{t}{t_f} = 0 \text{ , } \xi = \frac{R_c}{\Delta R} = 0$$

$$\text{at } \eta = \frac{t}{t_f} = 1 \text{ , } \frac{R_c}{\Delta R} = 1$$

and they are satisfied by (23). Thus (23 is the desired equation and may be written as (24), the range portion of the Polynomial Guidance Law.

$$R_c = \Delta R \left[ 1 - (1 - \frac{t}{t_f})^K \right] \qquad (24)$$

Note that since the slope of the function before the shift was zero at the origin, then after the shift at the corresponding point $\eta = 1$, $\xi = 1$, the slope remains zero (See Fig. 27 and 28) forcing the downrange velocity command ($R_c$) to be zero ($\frac{dR_c}{dt} = 0$) at $t = t_f$ (by the end of the flight).

A desireable requirement which should be imposed on both the altitude and range guidance laws is that the vertical velocity command $(\frac{dh_c}{dt})$ approach zero as the range approaches the desired value. It is seen that $(\frac{dh_c}{dR_{IM}} \rightarrow 0$ as $R_{IM} \rightarrow \Delta R$. Also $\frac{dR_c}{dt} \rightarrow 0$ as $t \rightarrow t_f$. Also the product $\frac{dR_c}{dt} \cdot \frac{dh_c}{dR_{IM}} \rightarrow 0$ as $t \rightarrow t_f$. If $R_c$ and $R_{IM}$ are equal than $\frac{dh_c}{dt} \rightarrow 0$ as $t \rightarrow t_f$.

$R_c$ and $R_{IM}$ should approach each other closely as $R \rightarrow \Delta R$ since if the guidance law and vehicle control loop together are functioning properly the vehicle would be following commands well enough to minimize the error between $R_c$ and $R_{IM}$. If this were not the case, then the guidance law could under no circumstances take the vehicle to the desired landing point.

4.2.4 (Continued)

Another desireable requirement affecting both laws which should also be imposed is that the altitude commanded velocity shall monotonically decrease as the downrange target is approached. This occurs if $\frac{dR_c}{dt}$ and $\frac{dh_c}{dR_{IM}}$ both monotonically decrease and if $R_c$ and $R_{IM}$ are nearly equal. For $N > 1$ in the altitude guidance law (19) and for $K > 1$ in the range guidance law (24) both derivatives monotonically decrease, therefore the commanded vertical velocity $\left(\frac{dh_c}{dt}\right)$ also monotonically decreases. It may be possible to have $N < 1$ so long as the $\frac{dR_c}{dt}$ decreases faster than $\frac{dh_c}{dR_{IM}}$ increases as $t \rightarrow t_f$. This flexibility in the choice of N would allow selection of a convex rather than a sagging trajectory if so desired (See Fig. 25) provided the aforementioned condition with regards to $R_c$ and $R_{IM}$ is met.

4.2.5 Generalized Range Portion of Polynomial Law

A similar procedure to that used in determining the range function may also be used to obtain a generalized function for range which could approximate a desired function. This may be accomplished by taking the highest ($K^{th}$) order term and adding terms of lower order (with coefficients) to obtain a polynomial which satisfies the trajectory requirements imposed. This function would take the form

$$\xi = 1 - \sum_{j=1}^{n} (1 - \eta)^{K_j}$$

$$\frac{R_c}{\Delta R} = 1 - \sum_{j=1}^{n} c_j \left(1 - \frac{t}{t_f}\right)^{K_j} \qquad (25)$$

$$\sum_{j=1}^{n} c_j = 1 \qquad \text{since at } t = 0,\ R_c = 0.$$

$K_j > 1$ for monotonically decreasing range velocity command as $t \rightarrow t_f$

4.2.6 Introduction of a Terminal Cuttoff for Polynomial Guidance at a Point Above the Lunar Surface

In order to perform a proper landing the distance from the vehicle CG to the landing gear pad ($h_{LG}$) must be taken into account in the guidance law. In addition, to allow for errors in guidance and perform a well controlled descent, it was decided to guide by means of the Polynomial Law only to a target hover point (with zero velocity) at some distance ($h_{SW}$ - measured from the landing gear pad) directly above the target. The descent

4.2.6 (Continued)

then be completed by a direct vertical descent to be discussed later. This reduces the altitude above aimpoint to less than h by the amount $h_{LG} + h_{SW}$ and the altitude portion of the generalized Polynomial Law (20) changes to the following

$$h_c = \left[ h - (h_{LG} + h_{SW}) \right] \sum_{i=1}^{P} d_i \left(1 - \frac{R_{IM}}{\Delta R}\right)^{N_i} \tag{26}$$

The Polynomial Guidance Law which is actually used, see equation (19) also changes accordingly

$$h_c = \left[ h - (h_{LG} + h_{SW}) \right] \cdot \left(1 - \frac{R_{IM}}{\Delta R}\right)^{N_i} \tag{27}$$

This is the altitude portion of the Polynomial Guidance Law in its final form. The range command equation remains unchanged. See equation (24)

In summary the Polynomial Guidance Law presented (as equations (24) and (27) constrains the following parameters via guidance commands as indicated:

1. A predetermined time of flight ($t_f$)
2. Predetermined vehicle downrange distance to target end point ($\Delta R$).
3. Predetermined vehicle altitude distance to target end point.
4. Predetermined nominal altitude vs. range profile for a given set of initial and boundary conditions.
5. Downrange and altitude velocity, as the aim point is approached, is constrained to approach zero (and decrease monotonically).
6. Introduction of a termination point at some fixed altitude above the lunar surface.

4.2.7 Terminal Descent to Touchdown After Completion of Polynomial Guidance Law Phase

Once the termination point for Polynomial Guidance has been attained with zero velocity conditions, it remains to proceed to touchdown. It has been found advantageous to employ a vertical descent with a constant vehicle attitude in order to limit attitude rates and angles off vertical at touchdown. A constant vehicle vertical acceleration command guidance law was implemented to complete the descent. This was later proven successful in computer runs which simulated the descent. The value for the acceleration in the guidance law was computed based on a given

4.2.7 (Continued)

initial velocity and final velocity desired and measured height above the surface. The formula used is:

$$A_c = \frac{\dot{H}_{TG}^{\,2} - \dot{H}_{IMf}}{2(H_{IMf} - h_{LG})} \qquad (28)$$

This method requires little computation since $A_c$ need be determined only once (although this is not a necessity), since fixed boundary conditions specify the complete formula.

This formula however may be more sensitive to error in velocity due to squaring of the quantities in the numerator resulting in errors in touchdown velocities. A constant velocity command may prove to be less influenced by error since it is directly proportional to velocity. Although the acceleration command law (28) has the advantage that a zero end velocity may be specified, an acceleration command law implies that there will be an acceleration at touchdown which can be translated into an additional force acting on the vehicle. Since engine cutoff prior to or at touchdown is a separate problem in itself and has been studied elesewhere (Ref. 17) it was felt that equation (28) would be satisfactory for the purposes of this study. Navigation and Guidance equipment errors were introduced in this study and the results of their affect on touchdown point conditions are presented in LMO-500-110 (Ref. 16).

## 4.2.8 LIST OF SYMBOLS USED FOR POLYNOMIAL LAW

| | |
|---|---|
| $a_c$ | constant acceleration command, ft/sec$^2$ |
| $c_j$ | generalized guidance law coefficient |
| $d_i$ | generalized guidance law coefficient |
| $h_{LG}$ | vertical distance from landing gear to C.G., ft. |
| $H_{SW}$ | predetermined altitude at termination of Polynomial Guidance, ft. |
| $h_c$ | altitude command, ft. |
| $h$ | altitude differential, ft. (See eq. 26) |
| $H_{IMf}$ | measured altitude at termination of Polynomial Guidance, ft. |
| $\dot{H}_{IMf}$ | measured vertical velocity at termination of Polynomial Guidance, ft/sec |
| $\dot{H}_{fG}$ | estimated vertical velocity at termination of Polynomial Guidance, ft/sec |
| $H_{TG}$ | desired vertical velocity at touchdown, ft/sec |
| $K_j$ | generalized guidance law again |
| $K$ | guidance law gain |
| $N_i$ | generalized guidance law gain |
| $N$ | guidance law gain |
| $n$ | positive integer |
| $p$ | positive integer |
| $R_c$ | range command, ft. |
| $R_{IM}$ | measured (or estimated or computed) range, ft. |
| $\Delta R$ | Total desired range, ft. |
| $t$ | time, sec. |
| $t_f$ | total time of flight under Polynomial Guidance, sec. |

4.2.8 LIST OF SYMBOLS USED FOR POLYNOMIAL LAW (Continued)

$\eta$ non-dimensional guidance law independent variable $(\frac{t}{t_f})$

$\xi$ non-dimensional guidance law independent variable $(\frac{R_c}{\Delta R})$

$\lambda$ non-dimensional guidance law dependent variable $(\frac{h_c}{\Delta h})$

$\mu$ non-dimensional guidance law variable $(\frac{R_{IM}}{\Delta R})$

## 5.0 REFERENCES


1. GAEC, LEM-IOM-62-47H, 18 December 1962, Rathke, C.W., "Technical Data Review"

2. GAEC, L250-M03-2, January, 1963, Rathke, C.W., "Technical Priority Efforts, Prime Responsibility for"

3. GAEC, LMO-540-41, April, 1963, Stern, E., and Catenaro, R., "Automatic Mission Study"

4. NASA TN D-772, April, 1961, L.S. Cicolani - Ames Research Center, "Trajectory Control in Rendezvous Problems Using Proportional Navigation.

5. GAEC, LMO-500-138, 3 January 1964, A. Bierman, GAEC Dynamic Analysis, "Automatic Hover-to-Touchdown Mission Using Modified Line of Sight Proportional Navigation"

6. GAEC, LMO-500-83, 9 September 1963, A. Bierman, GAEC Dynamic Analysis, "Mathematical Description of IBM 7094 Digital Program Used for Final Powered Descent and Automatic Lunar Landing Guidance Studies"

7. GAEC, LMO-540-36, 6 April 1963, R. Catenaro, GAEC Systems Analysis, "Study Request Memo SRM-3 to Dynamic Analysis. Determination of Feasibility of Various Guidance Laws for the Descent Phase of the LEM Automatic Mission; with Consideration Given to Important Navigation and Guidance and Flight Control Systems Capabilities and Errors"

8. GAEC, LMO-540-91, 3 July 1963, R. Catenaro, Systems Analysis, "Some General Comments on the Requirements and Constraints Applied to the Generation of a Guidance Law to Automatize and Semi-Automatize the "Hover" to Touchdown Maneuver"

9. GAEC, LED-540-2, 29 April 1963, R. Catenaro, Systems Analysis, "Preliminary Report on Automatic LEM Mission Study," (CONFIDENTIAL)

10. GAEC, PDM-323A-88, 11 September 1962, H. Sperling, Dynamic Analysis, "Preliminary Study of the Proportional Navigation Phase of the LEM Powered Lunar Descent"

11. GAEC, LMO-500-109, 16 December 1963, H. Sperling/M. Rimer, Dynamic Analysis, "Derivation of a Guidance Control Technique for Automatic Lunar Landing Hover-to-Touchdown"

5.0 REFERENCES (Cont'd)


12. GAEC, LMO-500-99, October, 1963, A. Bierman, "Utilization of Radar Altimeter During Final Power Descent," (CONFIDENTIAL)

13. GAEC, LMO-500-37 (Revision B), A. Nathan, "Recommended LEM $\Delta$V Budget," (CONFIDENTIAL)

14. GAEC, LED-540-3, July, 1963, "Back-up Guidance Requirements," (CONFIDENTIAL)

15. GAEC, LMO-500-76, July 1963, A. Bierman, "Line-of-Sight Proportional Navigation Equations for Planar Flight"

16. GAEC, LMO-500-110, H. Sperling/M. Rimer, "Analysis and Performance of an Automatic Lunar Landing System Including a Guidance and Navigation Equipment Error Study"

17. GAEC, LMO-500-136, 12 February 1964, M. Rimer/H. Sperling, Dynamic Analysis, F. Wood, Systems Simulation, "Analysis of LEM Lunar Landing Touchdown Conditions Derived from GAEC Simulator Results"

APPENDIX A

DETAILED DESCRIPTION OF EQUATIONS IN HOVER TO TOUCHDOWN SIMULATION PROGRAM USING PROPORTIONAL NAVIGATION LAWS

(Includes Symbol Definitions)

## General

Figure 29 is a detailed block diagram showing the basic equations used in the program to perform the functions as previously described and illustrated in Figure 2 discussed in Section 3.4. The interflow of signals between blocks is also shown in greater detail. A discussion follows which will describe the computations and equations involved in each block of Figure 3, and the major assumptions made will be indicated. See following list and figures for definition of terms used.

## Attitude and Engine Dynamics

These equations describe the change in vehicle inertia (I) as a function of fuel burn off rate. They also describe the vehicle pitch attitude ($\theta$) and attitude rate ($\dot{\theta}$) response to commands. The response is based on a linear second order approximation of the stabilization and control system with no consideration given to vehicle unbalance. A calculation is made to estimate the maximum vehicle pitch rate ($\dot{\theta}_{EXT}$) between simulation computer sampling intervals. This representation is based on the linear second order approximation of vehicle dynamics. The engine thrust and thrust rate output responses are described by a linear first order lag system.

## Translational Dynamics

The block of equations expresses the mass (m) and mass rate ($\dot{m}$) of change of the vehicle, based on a constant $I_{sp}$. The vehicle radial acceleration ($\ddot{r}_a$) and the central angular acceleration ($\ddot{\phi}$) as the vehicle translates are expressed in terms of the thrust magnitude and direction and the varying lunar g field. This g variation is expressed as an inverse square law relationship of distance between the LEM and the lunar mass CG. The assumption that vehicle attitude and thrust vector are rigidly fixed with respect to each other is employed (elimination of trim gimbal simulation) in order to simplify vehicle representation. Radial and central angular position and rates are determined by integration of vehicle translational accelerations.

## Transformation of Inertial Coordinates (See Figure 4)

This computation results in generation of the actual gravity acceleration acting on the vehicle based on actual vehicle position. In addition, actual rectangular components of the gravity acceleration vector are determined. (The term "actual" refers to expected position of the vehicle based on physical laws as opposed to measured position of the vehicle based on sensor navigational measurements.)

APPENDIX A (Cont'd)

Actual Thrust Velocity Increment

The incremental change in vehicle velocity due to thrust is determined by obtaining the difference of previous and present velocity data and removing the component which was due to the affect of gravity. This results in the incremental velocity change due to thrust only.

IMU Sensed Thrust Velocity Increment

This block determines the velocity increments that would be measured by and IMU which has bias errors ($e_x$, $e_y$) and platform angle errors introduced in an actual platform angle ($\theta_p$).

Navigation (Integration)

IMU sensed velocity increments due to thrust ($\Delta \dot{X}_{TS}$, $\Delta \dot{Z}_{TS}$) are added to previously updated velocity data ($\dot{X}_{Mc}$, $\dot{Z}_{Mc}$ derived from radar measurements) to this is added computed gravitational incremental velocity which is not sensed. (This calculation is based on the assumption that the value of g changes linearly from the previous computational period to the next so that the expected value of g may be predicted. Present g cannot be calculated directly since the new vehicle position ($V_M$) is not yet determined.) The summation of the above quantities will result in total measured vehicle velocity ($\dot{X}_M$, $\dot{Z}_M$). Rectangular position and velocity data $X_m$, $Z_m$, $\dot{X}_m$, $\dot{Z}_m$) is converted (See Figure 4) to data referenced to a cylindrical coordinate system ($r_m$, $\phi_m$, $\dot{r}_m$). In addition horizontal velocity ($V_{tan}$, in a direction parallel to the inertial X axis) is computed.

Gravity Computation for Navigation

This computation is based on measured vehicle radial distance from the moon's CG based on the inverse square law. The radial distance is determined from the navigation computations.

Guidance Law (In Addition See Figure 5)

This block consists first of computation of preliminary relative parameters between the LEM and target point prior to the guidance computation. These quantities are: relative central angle ($\phi_{TG}$) relative range ($R_{SL}$), rectangular coordinates of range ($X_{SL}$, $Z_{SL}$), LOS angle ($\psi$) based on navigation (measured)inputs. Flight path angle ($\gamma$) is computed based on radar data or on IMU data only. From the above data the lead angle (L) is determined.

The guidance computation is then performed based on both the Cicolani Guidance Law and the modification of the Proportional Guidance Law as previously discussed in Section 3.2 and 3.3. The guidance law outputs are acceleration command components along the velocity vector ($\dot{V}_c$) and normal to the velocity vector ($\dot{N}_c$). The total commanded acceleration magnitude ($A_c$) and direction with respect to the velocity vector ($\alpha$) is computed and converted to vehicle referenced thrust and attitude commands.

APPENDIX A (Cont'd)

Lunar Surface (In Addition See FIgure 5)

The computation of the lunar surface inertial coordinates directly below the LEM as a function of LEM central angle ($\phi_a$) is performed either for a spherical moon or for a moon with a linear inclined surface. The surface radius ($R_G$) is a constant as a function of central angle for a spherical moon and vehicle altitude may be determined by a subtraction of this constant radius from present vehicle radius ($r_a$). For the linearly inclined surface, the surface point coordinates ($Z_G$, $X_G$) vary as a function of target central angle ($\phi_T$) and slope of the incline. The inclined surface is assumed to pass through the preselected target point making an angle of $\alpha_T$ with the local horizontal. The lunar surface radial distance ($R_G$) is computed from the lunar surface rectangular coordinates, and then the altitude above the lunar surface is determined by subtraction of surface radius from vehicle radius ($r_a$) as previously. The Initial Set-up Calculation is used to insert the target point coordinates through which the inclined surface must pass.

Radar

This block of equations enables computations of the altitude ($h_R$), altitude rate ($\dot{h}_R$), and horizontal velocity ($V_H$) which would result from resolved radar measurements. An intermediate step in this determination is the computation of the intermediate quantities for altitude ($h^*$), altitude rate ($\dot{h}^*$), and horizontal rate. These quantities reflect only the introduction of platform angular error ($\Theta_p$) off the inertial reference axes which are introduced in the resolution using perfect radar data. To introduce radar sensitivity or bias errors, it is necessary first to decide which is the larger error and second to use the larger one in the determination of the total radar error. This error is added to the intermediate quantity to produce a simulation of what the radar would measure. The computational equations are accurate for antenna beams whose pointing directions are close to the local vertical.

Update Target (In Addition See Figure 5)

Employing inertial and radar navigational position data ($r_m$, $X_m$, $Z_m$) and altitude measurements ($h_r$), the surface coordinates directly below the LEM along its local vertical are first calculated ($R_{GM}$, $X_{GM}$, $Z_{GM}$). Using the central angle of the target point ($\phi_T$) and coordinates of the surface directly below the LEM along its local vertical ($R_{GM}$, $X_{GM}$, $Z_{GM}$) for two successive navigation computations, the most recent target radius ($r_T^{**}$) is determined. The target radius $r_T$ is determined by averaging the previous $r_T$ with the most recent ($r_T^{**}$) calculation. The coordinates of the target points ($X_T$, $Z_T$) are also computed.

Update Velocity

The vehicle velocity measured components ($\dot{X}_{Mc}$, $\dot{Z}_{Mc}$) are obtained by resolving vertical and horizontal components obtained from the radar block along the X and Z axes. These velocities are utilized by the navigation block

APPENDIX A (Cont'd)

of equations to update its velocity computations.

Nomenclature

The following terminology is used throughout this report for the Proportional Navigation Law, unless otherwise indicated.

| SYMBOL | DEFINITION |
|---|---|
| $a$ | acceleration ($ft/sec^2$) |
| $a_2$, $a_3$ | constants |
| $e$ | accelerometer bias error, adjusted for sampling rate (ft/sec) |
| FL | RCS torque |
| $g$ | acceleration due to lunar gravitational field ($ft/sec^2$) |
| $h$ | altitude above lunar surface (ft) |
| $I$ | principal moment of inertia about LEM y-axis ($slug\text{-}ft^2$) |
| $I_{sp}$ | specific impulse of main engine fuel (sec) |
| $K$ | guidance law constant |
| $K_m$ | modulator gain |
| $K_R$ | rate gyro gain |
| $k_1$, $k_2$ | radar altitude scale factor and bias errors, respectively |
| $k_3$, $k_4$ | radar altitude rate scale factor and bias errors, respectively |
| $k_5$, $k_6$ | radar horizontal velocity scale factor and bias errors, respectively |
| $L$ | lead angle; the misalignment of the velocity vector from the line-of-sight direction (See Figure 6) |
| LOS | Line-of-sight from LEM to target (See Figure 6) |
| $m$ | mass (slugs) |
| $N$ | normal to the velocity vector in a 90° CCW direction (See Figure 6) |
| $R_o$ | mean radius of lunar surface (ft) |
| $R_G$ | actual radius of lunar surface (ft) at a particular central angle |

APPENDIX A (Cont'd)

| SYMBOL | DEFINITION |
|---|---|
| $R_{GM}$ | radius of lunar surface determined from inertial measurements and radar measurements |
| $R_{sl}$ | slant range distance from LEM to target (ft) |
| $r$ | radial distance from moon center (ft) |
| $\Delta r_T$ | ALTITUDE OF TARGET SURFACE above reference surface (ft) |
| $S$ | guidance law constant |
| $T$ | thrust (lbs) |
| $T_f$ | remaining time of flight computed from $T_{f_o} - t$, used in modified law |
| $T_{f_o}$ | preselected initial time of flight,used in modified law (sec) |
| $t$ | time (sec) |
| $t_k$ | time scale for IMU/radar/spacecraft computer calculations (sec) |
| $\Delta t_k$ | $t_k - t_{k-1}$ |
| $t_n$ | time scale for "real world" calculations (sec) |
| $\Delta t_n$ | $t_n - t_n - 1$ |
| $V$ | inertial velocity (ft/sec), also forms one axis of ($\overline{V}$, $\overline{N}$) coordinate system (See Figure 6) |
| $V_{tan}$ | inertial horizontal velocity component determined from navigation loop (computation can also use both radar and inertial inputs) |
| $V_H$ | horizontal velocity component determined from radar and including platform angle error. |
| $x$, $z$ | inertial selenocentric rectangular coordinates such that z-axis lies along initial radius to LEM and the x-axis 90° clockwise to the z-axis. The plane of x, z is determined by the initial LEM and target position vectors (ft) (see figure 4) |
| $x_{sl}$, $z_{sl}$ | the target coordinates with respect to LEM local horizontal and vertical components, respectively (ft), (See figure 6) |

APPENDIX A (Cont'd)

| SYMBOL | DEFINITION |
|---|---|
| $\Delta \dot{X}_{Ta}$, $\Delta \dot{Z}_{Ta}$ | actual change in velocity component due to thrust |
| $\Delta \dot{X}_{Ts}$, $\Delta \dot{Z}_{Ts}$ | sensed change in velocity component due to thrust |
| $\alpha$ | angular orientation of velocity vector with respect to command thrust acceleration (See Figure 6) |
| $\alpha_T$ | inclination of lunar surface with respect to local horizontal at target |
| $\gamma$ | flight path angle measured to LEM local horizontal (see Figure 4) |
| $\theta$ | attitude angle measured with respect to LEM local horizontal (see Figure 4) |
| $\dot{\theta}_{EXT}$ | approximate maximum pitch rate calculated between simulation computer sampling instants based on 2nd order linear system |
| $\theta_p$ | platform pitch angle measured from inertial reference x-axis to platform x-axis in the CW direction. The presence of this angle will introduce a measurement error in the inertial and radar systems |
| $\mu$ | lunar gravitational constant ($ft^3/sec^2$) |
| $\tau$ | time constant for main engine (sec) |
| $\phi$ or $\varphi$ | central angle measured with respect to z-axis (see Figure 4) |
| $\phi_{TG}$ or $\varphi_{TG}$ | central angle between instantaneous LEM position and target (see Figure 4) |
| $\psi$ | LOS angle measured with respect to LEM local horizontal (see Figure 6) |
| $\omega$ | damped natural frequency of RCS loop |

## SUBSCRIPTS AND SUPERSCRIPTS

Unless otherwise noted a subscript is indicated

| | |
|---|---|
| a | actual value |
| c | commanded value |
| G | with respect to lunar surface |
| M | measured or computed value |
| $M_c$ | transformed and corrected value |
| T | with respect to target |
| x | component associated with inertial x-axis |
| z | component associated with inertial z-axis |
| o | initial value |
| R | from radar |
| H | horizontal component |
| S | sensed value |

Lack of a subscript generally indicates a parameter associated with the LEM vehicle.

A dot ( $\dot{}$ ) above a variable indicates differentiation with respect to time.

| | |
|---|---|
| *(superscript) | radar determination excluding radar error but including inertial measurement errors. |
| **(superscript) | most recent calculation of the variable (with no averaging) |

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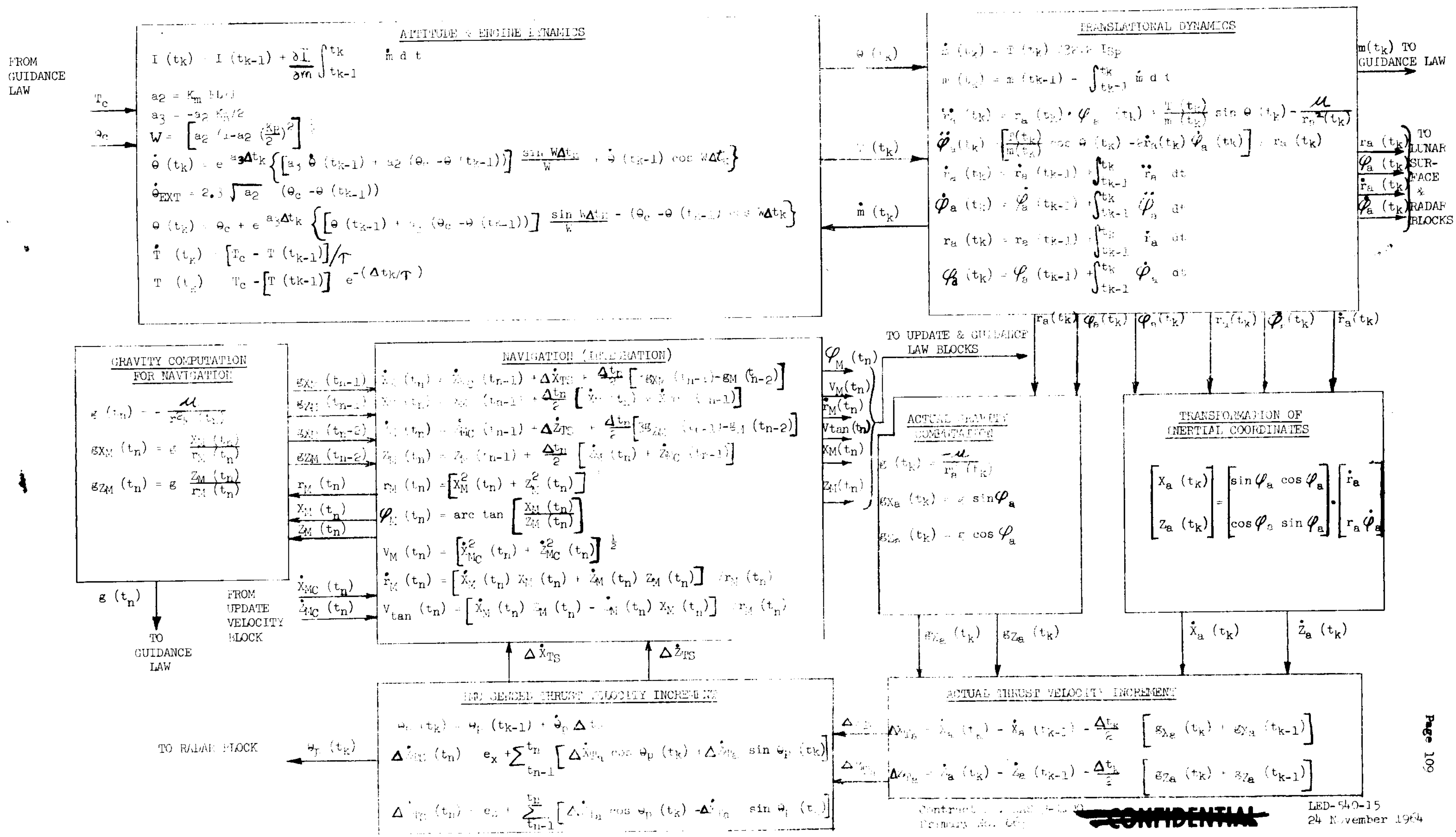
FIGURE 29

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GUIDANCE ANALYSIS EQUATIONS ~~CONFIDENTIAL~~

FIGURE 29

FROM TRANSLATION DYNAMICS BLOCK: $r_a(t_n)$, $\varphi_a(t_n)$

## LUNAR SURFACE

FOR INCLINED SURFACE

$$Z_G(t_n) = \frac{Z_T - X_T \tan(\alpha_T - \varphi_T)}{1 - \tan\varphi_a(t_n)\tan(\alpha_T - \varphi_T)}$$

$$X_G(t_n) = Z_G(t_n)\tan\varphi_a(t_n)$$

$$R_G(t_n) = \left[X_G^2(t_n) + Z_G^2(t_n)\right]^{\frac{1}{2}}$$

$$h_a(t_n) = r_a(t_n) - R_G(t_n)$$

FOR SPHERICAL MOON

$$R_G(t_n) = R_o$$

$$h_a(t_n) = r_a(t_n) - R_o$$

INITIAL SET-UP CALCULATION

$$r_T = R_o + \Delta r_T$$

$$X_T = r_T \sin\varphi_T$$

$$Z_T = r_T \cos\varphi_T$$

FROM NAV BLOCK: $\varphi_M(t_n)$, $X_M(t_n)$, $Z_M(t_n)$, $\dot r_M(t_n)$, $V_{tan}(t_n)$, $V_M(t_n)$

FROM GRAVITY BLOCK: $g(t_n)$

FROM TRANSLATION DYNAMICS BLOCK: $m(t_n)$

## GUIDANCE LAW

$$\varphi_{TG} = \varphi_T - \varphi_M(t_n)$$

$$R_{SL} = \left\{\left[X_T(t_n) - X_M(t_n)\right]^2 + \left[Z_T(t_n) - Z_M(t_n)\right]^2\right\}^{\frac{1}{2}}$$

$$X_{SL} = r_T(t_n)\sin\varphi_{TG}$$

$$Z_{SL} = r_T(t_n)\cos\varphi_{TG} - r_M(t_n)$$

$$\psi = \arctan(Z_{SL}/X_{SL})$$

$$\gamma = \arctan\left[\dot h_R(t_n)/V_H(t_n)\right] \text{ (WITH RADAR DATA)}$$

$$\gamma = \arctan\left[\dot r_M(t_n)/V_{tan}(t_n)\right] \text{ (WITH IMU DATA ONLY)}$$

$$L = \psi - \gamma$$

$$\dot V_C = \frac{V_M^2(t_n)}{R_{SL}}\left[(S-2)\cos L - \left(S - \frac{K+1}{K}\right)\frac{\sin L}{L}\right] + g(t_n)^{***}\sin\gamma$$

$$\dot N_C = \frac{S V_M^2(t_n)}{R_{SL}}\sin L + g(t_n)^{***}\cos\gamma$$

$$a_c = (\dot V_C^2 + \dot N_C^2)^{\frac{1}{2}}$$

$$\alpha = \arctan(\dot N_C/\dot V_C)$$

$$\theta_C = \alpha + \gamma$$

$$T_C = m\,a_c$$

$T_C$, $\theta_C$ TO PITCH & ENGINE DYNAMICS BLOCK

*** If Modified LOS Law is used $g(t_n)$ becomes $\frac{V}{V_R}\,g(t_n)$ where $V_R = K\,R_{SL}\left(\frac{L}{\sin L}\right)/T_f$

$h_a(t_n)$, $h_R(t_n)$, $V_H(t_n)$, $r_T(t_n)$, $X_T(t_n)$, $Z_T(t_n)$

FROM IMU DYNAMICS: $\theta_p(t_n)$

FROM TRANSLATION DYNAMICS: $r_a(t_n)$, $\dot r_a(t_n)$, $\varphi_a(t_n)$

## RADAR

$$h^* = h_a(t_n)/\cos\theta_p(t_n)$$

$$\dot h^* = \dot r_a(t_n)\cos\theta_p + r_a(t_n)\dot\varphi_a(t_n)\sin\theta_p(t_n)$$

$$V_H^* = r_a(t_n)\dot\varphi_a(t_n)\cos\theta_p(t_n) - \dot r_a(t_n)\sin\theta_p(t_n)$$

| | |
|---|---|
| If $\lvert k_1 h^*\rvert \geq \lvert k_2\rvert$ | $h_R(t_n) = (1 + k_1)h^*$ |
| If $\lvert k_1 h^*\rvert < \lvert k_2\rvert$ | $h_R(t_n) = h^* + k_2$ |
| If $\lvert k_3 \dot h^*\rvert \geq \lvert k_4\rvert$ | $\dot h_R(t_n) = (1 + k_3)\dot h^*$ |
| If $\lvert k_3 \dot h^*\rvert < \lvert k_4\rvert$ | $\dot h_R(t_n) = \dot h^* + k_4$ |
| If $\lvert k_5 V_H^*\rvert \geq \lvert k_6\rvert$ | $V_H(t_n) = (1 + k_5)V_H^*$ |
| If $\lvert k_5 V_H^*\rvert < \lvert k_6\rvert$ | $V_H(t_n) = V_H^* + k_6$ |

FROM NAV BLOCK: $r_M(t_n)$, $X_M(t_n)$, $Z_M(t_n)$

## UPDATE TARGET

$$R_{GM}(t_n) = r_M(t_n) - h_R(t_n)$$

$$X_{GM}(t_n) = R_{GM}(t_n)\cdot X_M(t_n)/r_M(t_n)$$

$$Z_{GM}(t_n) = R_{GM}(t_n)\,Z_M(t_n)/r_M(t_n)$$

$$r_T^*(t_n) = \frac{\left[X_{GM}(t_n) - X_{GM}(t_{n-1})\right]\cdot Z_{GM}(t_n) - \left[Z_{GM}(t_n) - Z_{GM}(t_{n-1})\right]X_{GM}(t_n)}{\left[X_{GM}(t_n) - X_{GM}(t_{n-1})\right]\cdot\cos\varphi_T - \left[Z_{GM}(t_n) - Z_{GM}(t_{n-1})\right]\sin\varphi_T}$$

$$r_T(t_n) = \left[r_T^{**}(t_n) + r_T(t_{n-1})\right]/2$$

$$X_T(t_n) = r_T(t_n)\sin\varphi_T$$

$$Z_T(t_n) = r_T(t_n)\cos\varphi_T$$

$r_M(t_n)$, $X_M(t_n)$, $Z_M(t_n)$, $\dot h_R(t_n)$, $V_H(t_n)$

## UPDATE VELOCITY

$$\dot X_{MC}(t_n) = \left[\dot h_R(t_n)\,X_M(t_n) + V_H(t_n)\,Z_M(t_n)\right]/r_M(t_n)$$

$$\dot Z_{MC}(t_n) = \left[\dot h_R(t_n)\,Z_M(t_n) - V_H(t_n)\,X_M(t_n)\right]/r_M(t_n)$$

$\dot X_{MC}(t_n)$, $\dot Z_{MC}(t_n)$ TO NAV BLOCK

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